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ON PROGNOSIS OSCILLATIONS OF THE MULTILAYER STRUCTURES

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Abstract. In this paper we analyzed sound influence on a multilayer building structures with increasing sound insulation on the example of oscillations of a multilayer plate. An analytical approach for analyzing the data of oscillations has been introduced.

Keywords: multilayer building structures, increasing sound insulation, analytical approach for modelling.
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1 Introduction

High rates of development of construction equipment create the necessary prerequisites for the design and construction of buildings and other structures of elements that have significant strength and stability with low weight and small thickness (Long et al., 2017; Xue & Zhang, 2018; Cu et al., 2018; Wen et al., 2018; Pisal & Jangid, 2016; Zhang et al., 2016; Gong et al., 2017; Broujerdian et al., 2018). At the same time, the development of technology leads to the emergence of more powerful machines and to increasing number of vehicles, which leads to increasing of noise in populated areas, civil and industrial buildings. Acoustic improvement of premises becomes an actual problem of each design and construction of each building (Nayak et al., 2018; Long et al., 2016; Tran et al., 2017; Argilaga et al., 2019; Zhao et al., 2019). Framework solving this problem, the problem of the sound-insulating ability of the enclosing and supporting structures is first of all solved. To solve the problem, it is necessary to analyze the sound effect on the structure. Framework the paper, an analysis of this effect has been done, taking into account the possible multilayered structure. We will analyze this effect using the example of transverse oscillations of a multilayer plate (structure) due to the action of a plane sound wave perpendicular to the interface between the layers of the plate.

2 Method of solution

The oscillation of the plate is determined by solving the following wave equation

$$\frac{\partial^2 u\ (x,y,z,t)}{\partial\, t^2} = \frac{E}{\rho\ (1-\sigma)} \frac{\partial^2 u\ (x,y,z,t)}{\partial\, x^2} + \frac{E}{\rho\ (1-\sigma)} \frac{\partial^2 u\ (x,y,z,t)}{\partial\, y^2} + F\ (x,y,z,t)\,, \qquad (1)$$

where E is the modulus of elasticity; ρ is the density of the plate materials; σ is the Poisson ratio, $u\ (x,y,z,t)$ is the displacement of the points of the plate when it oscillates; $F(x,y,z,t)$ is the external impact (impact, sound wave, etc.); L_x, L_y and L_z are the dimensions of the plate in the directions indicated in the indices; x, y and z are spatial coordinates; t is time. Let us consider the case when the edges of the plate are rigidly fixed and there is no effect on it at the time of the beginning of its consideration. Then the boundary and initial conditions for equation (1) could be written in the following form

$$\left.\frac{\partial u\ (x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial u\ (x,y,z,t)}{\partial x}\right|_{x=L_x}=0,$$
$$\left.\frac{\partial u\ (x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial u\ (x,y,z,t)}{\partial y}\right|_{x=L_y}=0, \tag{2}$$
$$u(x,y,z)=0.$$

Now we solve the Eq.(1) using the method of averaging functional corrections (Sokolov, 1955). In the framework of this method, in order to obtain the first approximation of the desired function $u(x,y,z,t)$, replace it with the unknown average value α_1 in the right-hand side of the Eq.(1). Then the equation for the first approximation of the function $u(x,y,z,t)$ takes the form

$$\frac{\partial^2\ u_1\,(x,y,z,t)}{\partial\ t^2}=F\ (x,y,z,t)\,. \tag{3}$$

The solution of Eq. (3) is represented in the following form

$$u_1\,(x,y,z,t)=\int_0^t (t-\tau)\ F\ (x,y,z,\tau)\ d\,\tau. \tag{3a}$$

The average value of the function $u(x,y,z,t)$ is determined using the standard relation

$$\alpha_1=\frac{1}{L_xL_yL_z\Theta}\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} u_1\,(x,y,z,t)\ d\,z\,d\,y\,d\,z\,d\,t. \tag{4}$$

where Θ is the duration of observation of the oscillation of the considered plate. Substitution of the relation (3a) into (4) leads to the following result

$$\alpha_1=-\frac{1}{2L_xL_yL_z\Theta}\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} t^2F\ (x,y,z,\tau)\ d\,z\,d\,y\,d\,z\,d\,t \tag{4a}$$

The second-order approximation of the function $u(x,y,z,t)$ is determined by replacing it in the right-hand side of the Eq. (1) by the sum of the approximation of the previous order and the mean value of the desired approximation α_2, i.e. by the amount $\alpha_2+u(x,y,z,t)$. Then the equation for the second approximation of the function $u(x,y,z,t)$ takes the form

$$\frac{\partial^2\ u_2\,(x,y,z,t)}{\partial\ t^2}=\frac{E}{\rho\ (1-\sigma)}\frac{\partial^2\ u_1\,(x,y,z,t)}{\partial\ x^2}+\frac{E}{\rho\ (1-\sigma)}\frac{\partial^2\ u_1\,(x,y,z,t)}{\partial\,y^2}+F\ (x,y,z,t)\,, \tag{5}$$

Solution of the Eq. (5) could be written as

$$u_2\,(x,y,z,t)=\frac{E}{\rho\ (1-\sigma)}\int_0^t (t-\tau)\,\frac{\partial^2\,u_1\,(x,y,z,\tau)}{\partial\ x^2}d\,\tau+$$
$$+\frac{E}{\rho\ (1-\sigma)}\int_0^t (t-\tau)\,\frac{\partial^2\,u_1\,(x,y,z,\tau)}{\partial\,y^2}d\,\tau+\int_0^t (t-\tau)\ F\ (x,y,z,\tau)\ d\,\tau. \tag{5a}$$

The average value α_2 of the second-order approximation of the function $u(x, y, z, t)$ is determined using the standard relation (Sokolov, 1955)

$$\alpha_n = \frac{1}{L_x L_y L_z \Theta} \int_0^{\Theta} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left[u_n\left(x, y, z, t\right) - u_{n-1}\left(x, y, z, t\right)\right] \, d\, z \, d\, y \, d\, z \, d\, t, \qquad (6)$$

where n is the order of the required approximation. Substitution of relations $(3a)$ and $(5a)$ into (6) leads to the following result

$$\alpha_2 = -\frac{1}{2 L_x L_y L_z \Theta} \int_0^{\Theta} t \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \frac{E}{\rho\,(1-\sigma)} \frac{\partial^2 F\left(x, y, z, t\right)}{\partial\, x^2} \, d\, z \, d\, y \, d\, z \, d\, t -$$
$$-\frac{1}{2 L_x L_y L_z \Theta} \int_0^{\Theta} t \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \frac{E}{\rho\,(1-\sigma)} \frac{\partial^2\, F\left(x, y, z, \tau\right)}{\partial\, y^2} \, d\, z \, d\, y \, d\, z \, d\, t. \qquad (6a)$$

An analysis of the spatio-temporal distribution of the displacement of the plate points during its oscillation has been done analytically framework the second-order approximation by the method of averaging the functional corrections. The approximation is usually enough for qualitative analysis and obtaining some quantitative results. The results of the analytical calculations were verified by comparing them with the numerical results.

3 Discussion

In this section, we will analyze the space-time distribution of the displacement of the points of the plate when it vibrates under the action of a plane wave $F(x, y, z, t) = A.exp(k_z z - \omega t)$, where A is the amplitude of the wave, k_z is the projection of the wave number on the O_z axis, and w is the wave frequency. Fig. 1 shows the qualitative spatial distribution of the displacement of the points of the plate as a function of the coordinates x and y at a fixed time. Fig. 2 shows the qualitative spatio-temporal distribution of the displacement of the plate points as a function of the coordinate z and time t for fixed values of the x and y coordinates.

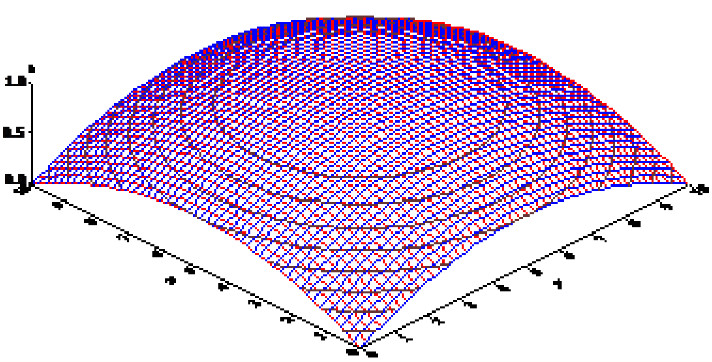

Figure 1: The qualitative spatial distribution of the displacement of the points of the plate as a function of the coordinates x and y at a fixed time

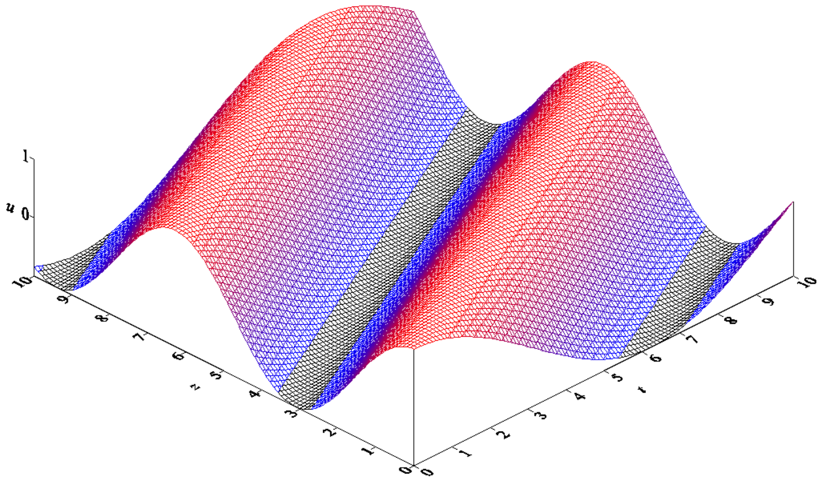

Figure 2: The qualitative space-time distribution of the displacement of the plate points as a function of the coordinate z and time t for fixed values of the coordinates x and y

4 Conclusion

In this paper, we propose an analytical approach for analyzing plate vibrations under the influence of external action. As an example of such an impact, sound impact is possible. The proposed analytical approach allows us to take into account the multilayeredness of the plate under consideration.

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