Advanced Mathematical Models & Applications
Vol.7, No.1, 2022, pp.93-103

RELIABILITY ANALYSIS OF A 3-UNIT PARALLEL SYSTEM WITH SINGLE MAINTENANCE FACILITY

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Abstract. This paper presents the reliability analysis of a cable plant system with three identical units operating in parallel, and with a single corrective maintenance strategy. A unit undergoes corrective maintenance upon failure. After each corrective maintenance, the unit works as good as new. The maintenance of the failed units in the system is carried out on the first-come-first-served basis, and in case of the second or third unit failure; it has to wait until the unit under maintenance is restored completely. The system goes in the completely failed state when all the three units are failed. The failures and repair pattern are taken as depicted in the maintenance data collected from the plant. Using the semi-Markov and regenerative processes, the reliability indices of the system such as mean time between failures, availability, expected busy period of the maintenance facility, expected number of corrective maintenances, and profit incurred are estimated. Some easy-fit software applications to the maintenance data are also included to support the analysis.

Keywords: availability, corrective maintenance, regenerative processes, reliability, semi-Markov processes.
AMS Subject Classification: 60K10.
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Received: 12 October 2021; Revised: 6 December 2021; Accepted: 20 December 2021; Published: 19 April 2022.

Notation

CM	corrective maintenance
MTBF	mean time between failures
pdf	probability density function
cdf	cumulative distribution function
LST	Laplace Stieltjes transform
LT	Laplace transform
λ	constant failure rate
γ	constant CM rate
g(t)	pdf of CM times
G(t)	cdf of CM times
Op	the unit is operative
Fr	the unit has failed and is under CM
FR	the unit has failed and is under CM from the previous state
FR'	the unit has failed and is under CM from the previous two states
Fw	the unit has failed and is waiting for CM
FW	the unit has failed and is waiting for CM from the previous state
Si	state i
q_{ij}	pdf from Si to Sj

q_{ij}	pdf from Si to Sj
Q_{ij}	cdf from Si to Sj
$f^*(s)$	Laplace transform of f(t)
$F^{**}(s)$	Laplace Stieltjes transform of F(t)
$f(t)^*g(t)$	Laplace convolution of f(t) and g(t)
$F(t)^{**}G(t)$	Laplace Stieltjes convolution of F(t) and G(t)
AV_0	availability
BP_0	expected busy period of the maintenance facility
NR_0	expected number of CM
P	profit

1 Introduction

Many researchers have contributed to the field of reliability modeling while analyzing complex industrial systems under different operating conditions and assumptions. Parashar and Taneja (2007) worked on reliability and profit evaluation of a hot standby PLC system based on a master slave concept and two types of repair facilities. Nilsson and Bertling (2007) performed cost analysis of wind power systems using condition monitoring. Mathew et al. (2011) presented reliability modelling and analysis of a two-unit continuous casting plant. Shakuntla et al. (2011) carried out reliability analysis of polytube industry using supplementary variable technique. Kumar and Kapoor (2013) discussed profit evaluation of a stochastic model on base transceiver system considering software-based hardware failures and congestion of calls. Padmavathi et al. (2014) worked on probabilistic analysis of a desalination plant with major and minor failures and shutdown during winter season. Rizwan et al. (2014) developed a general model for reliability analysis of a domestic wastewater treatment plant. Ahmad and Kumar (2015) performed profit analysis of a two-unit centrifuge system considering the halt state on occurrence of minor/major fault. Sharma and Kaur (2016) presented cost benefit analysis of a compressor standby system with preference of service, repair and replacement is given to recently failed unit. Adlakha et al. (2017) carried out reliability and cost-benefit analysis of a two-unit cold standby system used for communication through satellite with assembling and activation time. Naithani et al. (2017) discussed probabilistic analysis of a three-unit induced draft fan system with one warm standby with priority to repair of the unit in working state. Al Rahbi et al. (2019) worked on reliability analysis of a rodding anode plant in aluminum industry with multiple-units failure and single repairman. Kaur et al. (2020) performed reliability modelling of a gravity die casting system covering seven types of failure categories. Malhotra et al. (2021) presented reliability analysis a two-unit cold redundant system working in a pharmaceutical agency with preventive maintenance. Recently, Taj and Rizwan (2021) estimated the reliability indices of a complex industrial system using best-fit distributions for repair/restoration times. It is noted that the methodology for the analysis of complex industrial systems is already in place and is widely applied in the literature for different industrial system analysis by developing the case specific robust models. However, the novelty in the entire work lies in its application to a specific system as a potential case study from reliability perspective and obtaining the relevant reliability indices through modeling which reflects the system behavior. Therefore, the objective of the present work is to analyze a cable plant system from reliability perspective, that reflects the system performance in terms of reliability indices and profit incurred to the system. The outcomes of the analysis are useful for the maintenance team to develop the future maintenance strategies.

Thus, in this paper a detailed reliability analysis of a bunching system is carried out. This system is widely used in the manufacturing process of electrical cables. The function of this system is to combine seven copper wires together as a bunch. The system consists of three identical units. Being a parallel configuration system, the failed state is considered when all the three units fail. Real failure/maintenance data of the system depicts that a failed unit undergoes

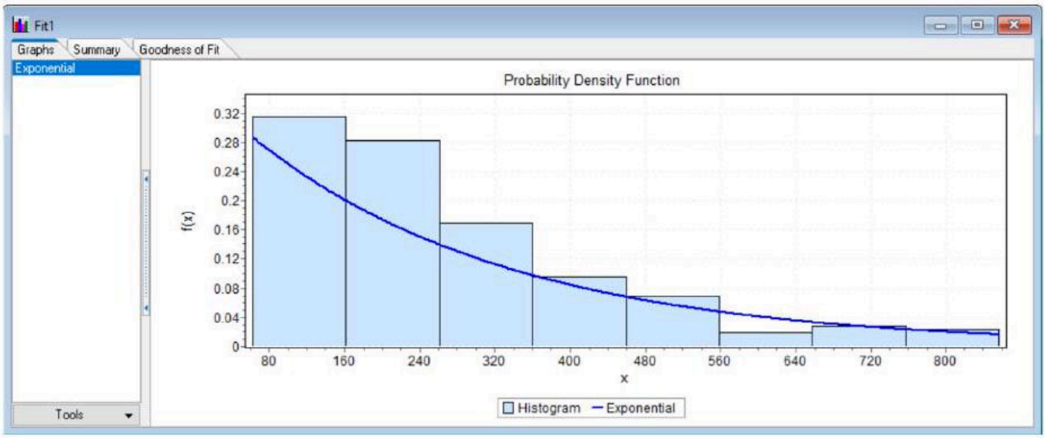

Figure 1: Distribution fitting - graphs (failure times)

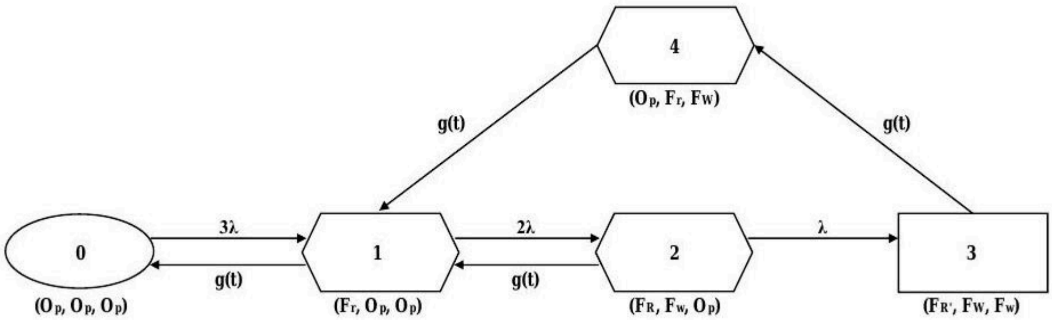

Figure 2: State-transition diagram

CM by a single maintenance facility, after which the unit regenerates. Important reliability indices (MTBF, availability, expected busy period of the maintenance facility, expected number of CM, profit incurred) of the system are estimated. Semi-Markov processes and regenerative processes are used for performing the analysis.

2 Model description

Following operating conditions and assumptions are considered:
- The system consists of three identical units operating in parallel.
- A unit undergoes CM upon failure.
- After each CM, the unit works as good as new.
- Only one maintenance at a time is carried out, and is on first-come-first-served basis.
- Failure times follow exponential distribution.
- CM times are assumed to follow arbitrary distribution.

Note: Using EasyFit software, the Kolmogorov-Smirnov test is applied for fitting exponential distribution to the failure times data. Result obtained is shown in figure 1.

The state-transition diagram of the system is shown in figure 2.

In figure 2, λ denotes the constant failure rate and g(t) denotes the pdf of CM times.

The states of the system are described below:

State 0 (S0): all the three units are operative.

State 1 (S1): one unit has failed and is under CM, the other two units are operative.

State 2 (S2): one unit has failed and is under CM from the previous state, one unit has failed

and is waiting for CM, one unit is operative.
State 3 (S3): one unit has failed and is under CM from the previous two states, one unit has failed and is waiting for CM from the previous state, one unit has failed and is waiting for CM.
State 4 (S4): one unit is operative, one unit has failed and is under CM, one unit has failed and is waiting for CM from the previous state.

Here,
S0, S1 and S4 are the regenerative states.
S2 and S3 are the non-regenerative states.
S3 is the failed state.

Note: The system is considered to be in the failed state when all the three units have failed.

3 Transition probabilities and mean sojourn times

Using the definition of transition probabilities $q_{ij}(t)$ (as defined in Taj and Rizwan (2021)), we get:

$$q_{01}(t) = 3\lambda e^{-3\lambda t}, \tag{1}$$
$$q_{10}(t) = g(t)e^{-2\lambda t}, \tag{2}$$
$$q_{12}(t) = 2\lambda e^{-2\lambda t}\overline{G}(t), \tag{3}$$
$$q_{11}^{2}(t) = \left(2\lambda e^{-2\lambda t} * e^{-\lambda t}\right) g(t), \tag{4}$$
$$q_{13}^{2}(t) = \left(2\lambda e^{-2\lambda t} * \lambda e^{-\lambda t}\right) \overline{G}(t), \tag{5}$$
$$q_{14}^{2,3}(t) = \left(2\lambda e^{-2\lambda t} * \lambda e^{-\lambda t} * 1\right) g(t), \tag{6}$$
$$q_{21}(t) = g(t)e^{-\lambda t}, \tag{7}$$
$$q_{23}(t) = \lambda e^{-\lambda t}\overline{G}(t), \tag{8}$$
$$q_{34}(t) = g(t) \tag{9}$$
$$q_{41}(t) = g(t), \tag{10}$$

where $G(t)$ denotes the cdf of repair times.

Using the definition of non-zero elements p_{ij} (as defined in Taj and Rizwan (2021)), we get

$$p_{01} = \frac{3\lambda}{3\lambda}, \tag{11}$$
$$p_{10} = g^*(2\lambda), \tag{12}$$
$$p_{12} = 1 - g^*(2\lambda), \tag{13}$$
$$p_{11}^{2} = 2\ g^*(\lambda) - 2\ g^*(2\lambda), \tag{14}$$
$$p_{13}^{2} = 1 - 2\ g^*(\lambda) + g^*(2\lambda), \tag{15}$$
$$p_{14}^{2,3} = 1 - 2\ g^*(\lambda) + g^*(2\lambda), \tag{16}$$
$$p_{21} = g^*(\lambda), \tag{17}$$
$$p_{23} = 1 - g^*(\lambda), \tag{18}$$
$$p_{34} = g^*(0), \tag{19}$$
$$p_{41} = g^*(0). \tag{20}$$

Following can be easily verified

$$p_{01} = 1, \tag{21}$$
$$p_{10} + p_{12} = 1, \tag{22}$$
$$p_{10} + p_{11}^{2} + p_{13}^{2} = 1, \tag{23}$$
$$p_{10} + p_{11}^{2} + p_{14}^{2,3} = 1, \tag{24}$$
$$p_{21} + p_{23} = 1, \tag{25}$$
$$p_{34} = 1, \tag{26}$$
$$p_{41} = 1. \tag{27}$$

Using the definition of mean sojourn time μ_i (as defined in Taj and Rizwan (2021)), we get

$$\mu_0 = \frac{1}{3\lambda}, \tag{28}$$
$$\mu_1 = \int_0^\infty \bar{G}(t)e^{-2\lambda t}dt, \tag{29}$$
$$\mu_2 = \int_0^\infty \bar{G}(t)e^{-\lambda t}dt, \tag{30}$$
$$\mu_3 = \int_0^\infty \bar{G}(t)dt, \tag{31}$$
$$\mu_4 = \int_0^\infty \bar{G}(t)dt. \tag{32}$$

Using the definition of unconditional mean time m_{ij} (as defined in Taj and Rizwan (2021)), following can be easily verified:

$$m_{01} = \mu_0, \tag{33}$$
$$m_{10} + m_{12} = \mu_1, \tag{34}$$
$$m_{10} + m_{11}^{2} + m_{13}^{2} = 2\mu_2 - \mu_1, \tag{35}$$
$$m_{10} + m_{11}^{2} + m_{14}^{2,3} = \mu_3 = \mu_4, \tag{36}$$
$$m_{21} + m_{23} = \mu_2, \tag{37}$$
$$m_{34} = \mu_3, \tag{38}$$
$$m_{41} = \mu_4. \tag{39}$$

4 Mean time between failures

Using simple probabilistic arguments and the definition of $\Psi_i(t)$ (as defined in Taj and Rizwan (2021)), we get

$$\Psi_0(t) = Q_{01}(t) * *\Psi_1(t), \tag{40}$$
$$\Psi_1(t) = Q_{10}(t) * *\Psi_0(t) + Q_{11}^{2}(t) * *\Psi_1(t) + Q_{13}^{2}(t), \tag{41}$$
$$\Psi_4(t) = Q_{41}(t) * *\Psi_1(t). \tag{42}$$

Taking Laplace Stielties transform (LST) of the above equation and solving for $\Psi_0{}^{**}(s)$, we obtain

$$\Psi_0{}^{**}(s) = \frac{N(s)}{D(s)} \tag{43}$$

The MTBF, given that the system started at the beginning of state 0 is given by

$$\mathrm{MTBF} = \lim_{s \to 0} \frac{1 - \Psi_0^{**}(s)}{s} = \frac{N}{D}, \tag{44}$$

where

$$N = 2\mu_2 - \mu_1 + \mu_0 \left(p_{10} + p_{13}^2\right). \tag{45}$$

$$D = p_{13}^2 \tag{46}$$

5 Availability

Using simple probabilistic arguments and the definition of $AV_i(t)$ (as defined in Taj and Rizwan (2021)), we get

$$AV_0(t) = M_0(t) + q_{01}(t) * AV_1(t), \tag{47}$$

$$AV_1(t) = q_{10}(t) * AV_0(t) + q_{11}^2(t) * AV_1(t) + q_{14}^{2,3}(t) * AV_4(t), \tag{48}$$

$$AV_4(t) = q_{41}(t) * AV_1(t). \tag{49}$$

where

$$M_0(t) = e^{-3\lambda t}. \tag{50}$$

Taking Laplace transform (LT) of the above equations and solving for $AV_0{}^*(s)$ we get

$$AV_0{}^*(s) = \frac{N_1(s)}{D_1(s)}. \tag{51}$$

In steady state, the availability of the system is given by

$$AV_0 = \lim_{s \to 0} s AV_0{}^*(s) = \frac{N_1}{D_1}. \tag{52}$$

where

$$N_1 = p_{10}\mu_0, \tag{53}$$

$$D_1 = p_{10}\mu_0 + \left(1 + p_{14}^{2,3}\right)\mu_4. \tag{54}$$

6 Expected busy period of the maintenance facility

Using simple probabilistic arguments and the definition of $BP_i(t)$ (as defined in Taj and Rizwan (2021)), we get

$$BP_0(t) = q_{01}(t) * BP_1(t), \tag{55}$$

$$BP_1(t) = W_1(t) + q_{10}(t) * BP_0(t) + q_{11}^2(t) * BP_1(t) + q_{14}^{2,3}(t) * BP_4(t), \tag{56}$$

$$BP_4(t) = W_4(t) + q_{41}(t) * BP_1(t), \tag{57}$$

where

$$W_1(t) = e^{-2\lambda t}\overline{G}(t), \tag{58}$$

$$W_4(t) = \overline{G}(t). \tag{59}$$

Taking LT of above equations and solving for $\mathrm{BP_0}^*(\mathrm{s})$ we obtain

$$\mathrm{BP_0}^*(\mathrm{s}) = \frac{\mathrm{N_2(s)}}{\mathrm{D_1(s)}}. \tag{60}$$

In steady state, the expected busy period of the maintenance facility is given by

$$\mathrm{BP_0} = \lim_{\mathrm{s}\to 0} \mathrm{sBP_0}^*(\mathrm{s}) = \frac{\mathrm{N_2}}{\mathrm{D_1}}, \tag{61}$$

where

$$\mathrm{N_2} = \mu_1 + \mathrm{p}_{14}^{2,3}\mu_4, \tag{62}$$

$\mathrm{D_1}$ is specified in equation (54).

7 Expected number of corrective maintenances

Using simple probabilistic arguments and the definition of $\mathrm{NR_i(t)}$ (as defined in Taj and Rizwan (2021)), we get:

$$\mathrm{NR_0(t)} = \mathrm{Q_{01}(t)} ** \{\mathrm{NR_1(t)} + 1\}, \tag{63}$$
$$\mathrm{NR_1(t)} = \mathrm{Q_{10}(t)} ** \mathrm{NR_0(t)} + \mathrm{Q}_{11}^{2}(\mathrm{t}) ** \{\mathrm{NR_1(t)} + 1\} + \mathrm{Q}_{14}^{2,3}(\mathrm{t}) ** \{\mathrm{NR_4(t)} + 1\}, \tag{64}$$
$$\mathrm{NR_4(t)} = \mathrm{Q_{41}(t)} ** \{\mathrm{NR_1(t)} + 1\}. \tag{65}$$

Taking LST of the above equations and solving for $\mathrm{NR_0}^{**}(\mathrm{s})$ we get

$$\mathrm{NR_0}^{**}(\mathrm{s}) = \frac{\mathrm{N_3(s)}}{\mathrm{D_1(s)}}. \tag{66}$$

In steady state, the expected number of corrective maintenances is given by

$$\mathrm{NR_0} = \lim_{\mathrm{s}\to 0} \mathrm{sNR_0}^{**}(\mathrm{s}) = \frac{\mathrm{N_3}}{\mathrm{D_1}}, \tag{67}$$

where

$$\mathrm{N_3} = 1 + \mathrm{p}_{14}^{2,3}, \tag{68}$$

$\mathrm{D_1}$ is specified in equation (54).

8 Profit analysis

The profit incurred to the system is given by the following equation:

$$\mathrm{P} = \mathrm{C_0AV_0} - \mathrm{C_1BP_0} - \mathrm{C_2NR_0}, \tag{69}$$

where
$\mathrm{C_0}$ is the revenue per unit up-time generated by the system;
$\mathrm{C_1}$ is the maintenance facility cost per unit time;
$\mathrm{C_2}$ is the cost per unit CM.

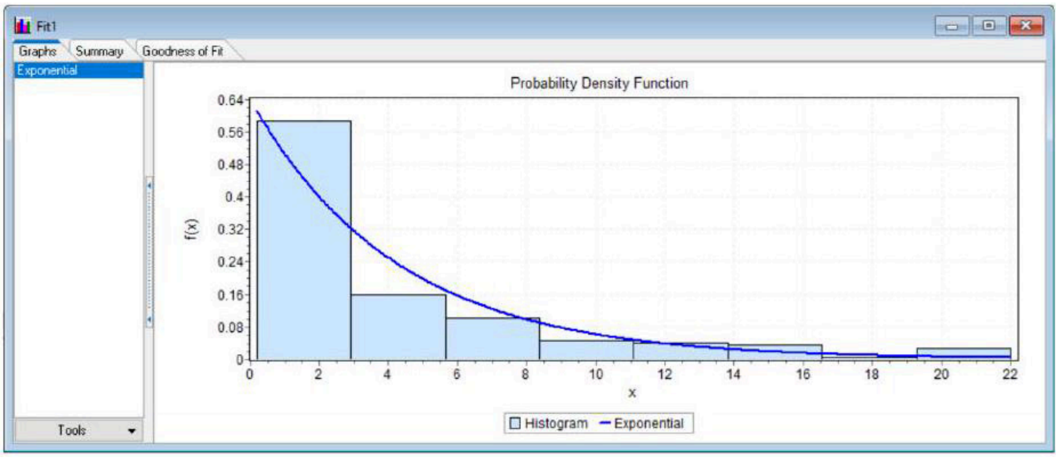

Figure 3: Distribution fitting - graphs (CM times)

Table 1: Estimated values of rates

Rate	Value
λ, constant failure rate	0.00364 per hour
γ, constant CM rate	0.23537 per hour

9 Particular case

For estimation purpose, let us assume that the CM times also follow exponential distribution, say:

$$g(t) = \gamma e^{-\gamma t}, \tag{70}$$

where γ denotes the constant CM rate.
Note: Using EasyFit software, the Kolmogorov-Smirnov test is applied for fitting exponential distribution to the CM times data. Result obtained is shown in figure 3.

The estimated values of constant failure rate and constant CM rate are given in table 1.

The values of various costs as collected from the operations/maintenance department of the industry are given in table 2.

Table 2: Values of various costs

Cost	Value
C_0, revenue per unit up-time generated by the system	2535 pounds/hour
C_1, maintenance facility cost per unit time	17 pounds/hour
C_2, cost per unit CM	188 pounds

Using the values from tables 1 and 2 in the expressions obtained in sections 1.4 - 1.8, the reliability indices of the system are evaluated which are shown in figures 4, 5 and 6.

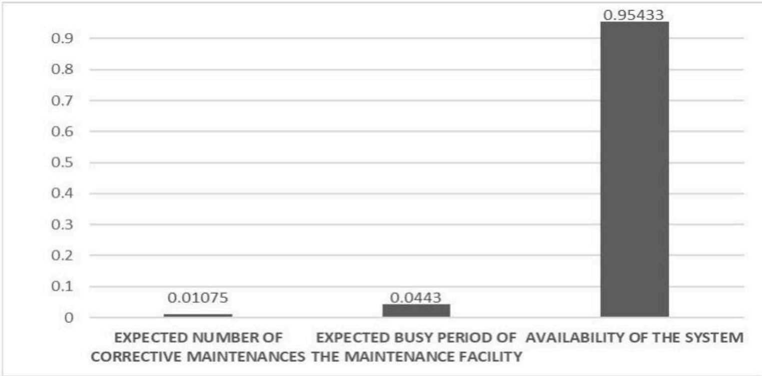

Figure 4: Expected no. of CM, Expected busy period of maintenance facility, System availability

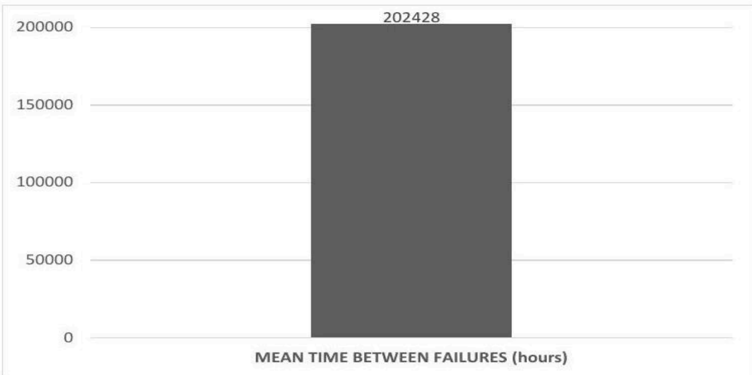

Figure 5: Mean time between failures

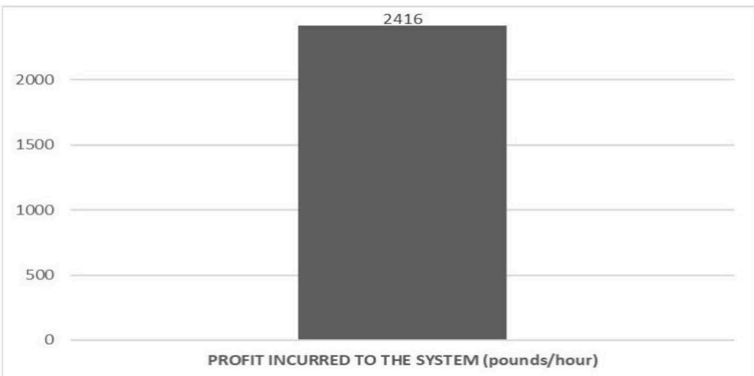

Figure 6: Profit incurred to the system

10 Conclusion

The outcomes of the analysis are measured in terms of the reliability indices such as mean time between failures of the system, expected availability of the system, expected busy period of the maintenance facility, expected number of corrective maintenances of the system, and profit incurred to the system. The application of easy-fit software suggests the suitability of the appropriate distribution to the maintenance data. The methodology can further be extended to study the reliability of systems with multiple units and mixed configurations, with identical or non-identical units, and for different failure/maintenance categories.

11 Acknowledgement

The authors would like to thank the reviewers for their valuable remarks, comments and advice that helped to improve the quality of the paper.

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