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EXPANSION METHOD FOR THE LOADED MODIFIED ZAKHAROV-KUZNETSOV EQUATION

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Abstract. Mathematical modeling of many physical process leads to nonlinear evolution equations. The investigation of traveling wave solutions of nonlinear partial differential equations plays significant role in the study of nonlinear physical phenomena. In this article, the generalized (G'/G) - expansion method using generalized wave transformation is applied to find exact traveling wave solutions of the loaded modified Zakharov-Kuznetsov(ZK) equation. The performance of the method is reliable, useful and gives more new general exact solutions than the existing methods. The travelling wave solutions are expressed by the hyperbolic functions, the trigonometric functions and the rational functions. The results emphasize the power of proposed methods in providing distinct solutions of different physical structures in nonlinear science. The graphical presentations are demonstrated for some of newly obtained solutions. It is shown that the proposed method provides a powerful mathematical tool for solving nonlinear wave equations in mathematical physics and engineering.

Keywords: loaded modified Zakharov-Kuznetsov equation, nonlinear evolution equations, expansion method, hyperbolic function, solitary wave.

AMS Subject Classification: 34A34, 34B15, 35Q51, 35J60, 35J66.

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1 Introduction

Nonlinear evolution equations (NLEE) appear in various fields of science and technology, such as fluid mechanics, plasma physics, optical fibers, biophysics, electricity, wave propagation in shallow water, high energy physics, biology, solid state physics, etc. One of the most important NPDE is the Zakharov–Kuznetsov (ZK) equation.

The Zakharov–Kuznetsov (ZK) equation is a very attractive model equation for the study of vortices in geophysical flows. The ZK equation appears in many areas of physics, applied mathematics and engineering. In particular, it shows up in the area of plasma physics (Das et al., 2007; Lin & Zhang, 2007; Mushtaq & Shah, 2005). The ZK equation governs the behavior of weakly nonlinear ion-acoustic waves in a plasma comprised of cold (Lu et al., 2008a,b; Biswas & Zerrad, 2010). In 1974, Zakharov and Kuznetsov formulated model equation for model the propagation of weakly nonlinear ion-acoustic waves in plasma, which includes cold ions and hot-isothermal electrons in a medium with a uniform magnetic field amplitude (Zakharov & Kuznetsov, 1974; Monro & Parkes, 1999).

The modified Zakharov–Kuznetsov (mZK) equation,

$$u_t + u^2 u_x + u_{xxx} + u_{xyy} = 0, \tag{1}$$

represents an anisotropic two-dimensional generalization of the Korteweg–de Vries equation and can be derived in a magnetized plasma for small amplitude Alfven waves at a critical angle to the undisturbed magnetic field, and has been studied by many authors because of its importance.

Many powerful and direct methods of nonlinear evolutions equations have been developed to find special solutions, such as, Weierstrass elliptic function method (Kudryashov, 1990), Jacobi elliptic function expansion method (Chen & Wang, 2005), tanh-function method (Malfliet, 1992), inverse scattering transform method Ablowitz & Segur (1991), Hirota method (Hirota, 1971), Backlund transform method (Rogers & Shadwick, 1982), exp-function method (He & Wu, 2006; **?**; Naher et al., 2011), truncated painleve expansion method (Kudryashov, 1991) and extended tanh-method (Abdou & Soliman, 2006; El-Wakil & Abdou, 2007) are used for searching the exact solutions.

Many authors have studied the modified ZK equation (Pelinovsky & Grimshaw, 1996; Sipcic & Benney, 2000; Shi et al., 2006; Zhao et al., 2006; Wazwaz, 2008; Tascan et al., 2009; Ali et al., 2020). The authors applied the asymptotic approach into modified ZK equation and found for the modified ZK equation that critical collapse in two dimensions is accompanied by damping both the momentumin and energy of the perturbed solitary waves and that slows down the rate of the singularity formation (Pelinovsky & Grimshaw, 1996), considered the two-dimensional solitary wave (lump) interactions and the formation of singularities in modified ZK equation considered the two-dimensional solitary wave (lump) interactions and the formation of singularities in modified ZK equation (Sipcic & Benney, 2000), obtained a class of approximate periodic solutions for modified ZK equation by using the homotopy analysis method considered the two-dimensional solitary wave (lump) interactions and the formation of singularities in modified ZK equation (Shi et al., 2006). The authors obtained many solitary waves and periodic waves and kink waves of modified ZK equation by using the theory of bifurcations of dynamical systems in (Zhao et al., 2006). Using the extended tanh method got new travelling wave solutions with solitons and periodic structures (Wazwaz, 2008). In Tascan et al. (2009), the first integral method was used to construct travelling wave solutions of modified ZK equation. Peng developed the extended mapping method to study the traveling wave solution for the modified ZK equation applied the exp-function method to construct generalized solitary and periodic solutions of modified ZK equation (Peng, 2009; Noor et al., 2010). In Naher & Abdullah (2012), authors applied the improved (G'/G)-expansion method to construct abundant new exact traveling wave solutions of modified ZK equation, employed the complex method to obtain the exact solutions of modified ZK equation (Yuan et al., 2013).

In this article, we consider the following the loaded modified ZK equation

$$u_t + u^2 u_x + u_{xxx} + u_{xyy} + \gamma(t)u(0,0,t)u_x = 0, \tag{2}$$

where $u(x,y,t)$ is an unknown function, $x \in R$, $y \in R$, $t \geq 0$, $\gamma(t)$ - is the given real continuous function.

We construct exact travelling wave solutions of the loaded modified ZK by (G'/G) - expansion method. The performance of this method is reliable and effective and gives the exact solitary wave solutions and periodic wave solutions. The traveling wave solutions obtained via this method are expressed by hyperbolic functions and the trigonometric functions. The graphical representations of some obtained solutions are demonstrated to better understand their physical features, including bell-shaped solitary wave solutions, singular soliton solutions and solitary wave solutions of kink type. This method presents a wider applicability for handling nonlinear wave equations.

It is known that the loaded differential equations contain some of the traces of an unknown function. In Kneser (1914); Lichtenstein (1931); Nakhushev (1995, 1983, 1976); Nakhushev & Borisov (1977); Nakhushev (1979), the term of "loaded equation" was used for the first time, the most general definitions of the loaded differential equation were given and also a detailed classifications of the differential loaded equations as well as their numerous applications were presented.

A complete description of solutions of the nonlinear loaded equations and their applications can be found in papers Baltaeva (2012); Kozhanov (2004); Hasanov & Hoitmetov (2021a,b); Khasanov & Hoitmetov (2021); Urazboev (2021); Yakhshimuratov & Matyokubov (2016).

2 Description of the generalized (G'/G) - expansion method

Consider nonlinear evolution equations with independent variables x, y and t is of the form

$$F(u, u_x, u_y, u_t, u_{xx}, u_{tt}, u_{yy}, u_{xy}, u_{xt}, u_{yt}...) = 0, \tag{3}$$

$u = u(x, y, t)$ is a unknown function, F is a polynomial in $u = u(x, y, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. Now we give the main steps of the (G'/G) - expansion method.

Step 1. By using traveling wave transformation

$$u(x, y, t) = u(\xi),\ \xi = px + qy - \varphi(t), \tag{4}$$

where $\dot{\varphi}(t)$ is the speed of the traveling wave. Now, using (4), the equation (3) is converted into an ordinary differential equation for $u = u(\xi)$:

$$P(u, u', u'', u''', ...) = 0, \tag{5}$$

where P is a polynomial of $u(\xi)$.

Step 2. Suppose the solution of (5) can be expressed by a polynomial in (G'/G) as follows:

$$u(\xi) = \sum_{j=0}^{m} a_j \left(\frac{G'}{G}\right)^j, \tag{6}$$

where $G = G(\xi)$ satisfies the following second order linear ordinary differential equation

$$G'' + \lambda G' + \mu G = 0, \tag{7}$$

while a_j, λ and μ are constants.

Step 3. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in (5). The coefficients of this polynomial can be obtained by solving a set of algebraic equations resulted from the process of using the method.

Step 4. Substitute (6) along with (7) into (5) and collect all terms with the same order of $G'(\xi)/G(\xi)$, the left-hand side of (5) is converted into a polynomial in $G'(\xi)/G(\xi)$. Next, equating the coefficients of each power of (G'/G) to zero, obtain a system of algebraic equations for a_j and $\varphi(t)$. Then, $u(\xi)$ can be restored by (6).

3 Exact solutions of the loaded modified equation of Zakharov-Kuznetsov

We will show how to find the exact solution of the loaded modified ZK equation using the (G'/G) - expansion method. For doing this, in (2), we use the following transformation

$$u(x, y, t) = u(\xi), \xi = px + qy - \varphi(t). \tag{8}$$

In (8), p, q are constants, $\varphi(t)$ is unknown function to be determined later and $\dot{\varphi}(t)$ gives the speed of the traveling wave. This transformation permits us converting equation (2) into an ordinary differential equation for $u = u(\xi)$:

$$-\dot{\varphi}u' + pu^2u' + p^3u''' + pq^2u''' + p\gamma(t)u(0, 0, t)u' = 0, \tag{9}$$

where $\gamma(t)$ is the given arbitrary real continuous function of t. Integrating (9) with respect to ξ once

$$-3\dot{\varphi}u + pu^3 + 3(p^3 + pq^2)u'' + 3\alpha(t)u = 0, \tag{10}$$

where $\alpha(t) = p\gamma(t)u(0,0,t)$ is the loaded term of eq (1).

We express the solution of equation (10) in the form of a polynomial in (G'/G) below

$$u(\xi) = \sum_{j=0}^{m} a_j \left(\frac{G'}{G}\right)^j, \tag{11}$$

$$G'' + \lambda G' + \mu G = 0. \tag{12}$$

Using (11) and (12), u^3 and u'' are easily derived to

$$u^3(\xi) = a_m^3 \left(\frac{G'}{G}\right)^{3m} + ..., \tag{13}$$

$$u''(\xi) = m(m+1)a_m \left(\frac{G'}{G}\right)^{m+2} + ... \,. \tag{14}$$

Considering the homogeneous balance between $u''$and u^3 in equation (10), based on (13) and (14) we required that $m = 1$. Taking into account considerations, the form of u is as following

$$u(\xi) = a_0 + a_1 \left(\frac{G'}{G}\right), \tag{15}$$

Now, if we let

$$Y = Y(\xi) = \frac{G'}{G},\ u(\xi) = a_0 + a_1 Y.$$

Then by the help of (12) we get

$$Y' = \frac{GG'' - G'^2}{G^2} = \frac{G(-\lambda G' - \mu G) - G'^2}{G^2} = -Y^2 - \lambda Y - \mu.$$

By result above and implicit difierentiation, one can derive the following two formulas

$$Y'' = 2Y^3 + 3\lambda Y^2 + (2\mu + \lambda^2)Y + \lambda\mu,$$

$$Y''' = -6Y^4 - 12\lambda Y^3 - (7\lambda^2 + 8\mu)Y^2 - (\lambda^3 + 8\lambda\mu)Y - (\lambda^2\mu + 2\mu^2).$$

Combining equations (11) and (12) then it results in a polynomial of powers of Y. Then, collecting all terms of same order of Y and equating to zero, yields a set of algebraic equations for $a_0,\ a_1,\ a_2,\ ...\ a_m$.

It is known that the solution of equation (12) is a linear combination of sinh and cosh, respectively, if $\Delta = \lambda^2 - 4\mu > 0$ or $\Delta = \lambda^2 - 4\mu < 0$. Without lost of generality, we consider the first case and therefore

$$G(\xi) = e^{-\frac{\lambda\xi}{2}}(A \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi + B \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi),$$

$$\frac{G'(\xi)}{G(\xi)} = -\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \frac{A \sinh \frac{\sqrt{\lambda^2-4\mu}}{2}\xi + B \cosh \frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{A \cosh \frac{\sqrt{\lambda^2-4\mu}}{2}\xi + B \sinh \frac{\sqrt{\lambda^2-4\mu}}{2}\xi},$$

where A and B are any constants.

We know the exact view of u^3

$$u^3(\xi) = a_0^3 + 3a_0^2 a_1 Y + 3a_0 a_1^2 Y^2 + a_1^3 Y^3, \tag{16}$$

using (15) and (12) u' and u'' are easily derived to

$$\begin{cases} u'(\xi) = -a_1Y^2 - a_1\lambda Y - a_1\mu, \\ u''(\xi) = 2a_1Y^3 + 3a_1\lambda Y^2 + (2a_1\mu + a_1\lambda^2)Y + a_1\lambda\mu. \end{cases} \tag{17}$$

By substituting (15)-(17) into equation (10) and collecting all terms with the same power of Y, the left-hand side of equation (10) is converted into another polynomial in Y.

$$\begin{aligned} &-3\dot{\varphi}(a_0 + a_1Y) + p(a_0^3 + 3a_0^2a_1Y + 3a_0a_1^2Y^2 + a_1^3Y^3) + 3(p^3 + pq^2)(2a_1Y^3 + 3a_1\lambda Y^2 + \\ &+(2a_1\mu + a_1\lambda^2)Y + a_1\lambda\mu) + 3\alpha(t)(a_0 + a_1Y) = 0, \end{aligned}$$

$$\begin{aligned} &(pa_1^3 + 6(p^3 + pq^2)a_1)Y^3 + (3pa_0a_1^2 + 9a_1\lambda(p^3 + pq^2))Y^2 + \\ &+(3pa_0^2a_1 + 6a_1(p^3 + pq^2)\mu + 3(p^3 + pq^2)a_1\lambda^2 - 3\dot{\varphi}a_1 + \\ &+3\alpha(t)a_1)Y + pa_0^3 + 3a_1\lambda\mu(p^3 + pq^2) - 3\dot{\varphi}a_0 + 3\alpha(t)a_0 = 0. \end{aligned} \tag{18}$$

Equating each coefficient of expression (18) to zero, yields a set of simultaneous equations for a_0, a_1 and $\varphi(t)$ as following:

$$Y^3 : pa_1^3 + 6(p^3 + pq^2)a_1 = 0,$$

$$Y^2 : 3pa_0a_1^2 + 9a_1\lambda(p^3 + pq^2) = 0,$$

$$Y^1 : 3pa_0^2a_1 + 6a_1(p^3 + pq^2)\mu + 3(p^3 + pq^2)a_1\lambda^2 - 3\dot{\varphi}a_1 + 3\alpha(t)a_1 = 0,$$

$$Y^0 : pa_0^3 + 3a_1\lambda\mu(p^3 + pq^2) - 3\dot{\varphi}a_0 + 3\alpha(t)a_0 = 0,$$

$$\begin{cases} a_1^2 = -6(p^2 + q^2), \\ a_0 = -\frac{3\lambda(p^2+q^2)}{a_1}, \\ \dot{\varphi} = \frac{p}{2}(p^2 + q^2)(4\mu - \lambda^2) + \alpha(t), \\ \frac{p}{2}(p^2 + q^2)(4\mu - \lambda^2) + \alpha(t) - \dot{\varphi} = 0. \end{cases}$$

By solving these equations, we obtain the followings

$$\begin{cases} a_0 = \pm\frac{\lambda(\sqrt{6(p^2+q^2)})}{2}i, \quad a_1 = \pm\sqrt{6(p^2 + q^2)}i, \\ \varphi(t) = \frac{p}{2}(p^2 + q^2)(4\mu - \lambda^2)t + p\int_0^t \gamma(\tau)u(0, 0, \tau)d\tau + \varphi^0, \end{cases} \tag{19}$$

λ, μ, p, q and φ^0 are arbitrary constants. Using (8) and (19), expression (15) can be rewritten as

$$u(\xi) = \pm\frac{\lambda(\sqrt{6(p^2 + q^2)})}{2}i \pm \sqrt{6(p^2 + q^2)}iY = \pm\frac{\sqrt{6(p^2 + q^2)}i}{2}\left(2\frac{G'(\xi)}{G(\xi)} + \lambda\right). \tag{20}$$

Substituting the general solutions of equation (11) into (20) with $\Delta = \lambda^2 - 4\mu > 0$, we get the solution of the loaded modified ZK equation (2)

$$u(\xi) = \pm\frac{\sqrt{6(p^2 + q^2)}i}{2}\sqrt{\lambda^2 - 4\mu}\frac{A\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + B\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{A\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + B\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}, \tag{21}$$

where A, B are an arbitrary constants and $\xi(x, y, t) = px + qy - \varphi(t)$. For define $\varphi(t)$ in (19) we seek $\xi(0, 0, t)$ as follows

$$\xi(0, 0, t) = \sum_{j=1}^{n} \alpha_jt^j. \tag{22}$$

In (22), α_j are unknown constants which should be defined. Differentiating the equality (22) with respect to t and we obtain

$$\xi_t' = \sum_{j=1}^{n} j\alpha_j t^{j-1}. \tag{23}$$

On the other hand, we have

$$\xi_t' = -\frac{p}{2}(p^2+q^2)(4\mu-\lambda^2) - p\gamma(t)u(0,0,t). \tag{24}$$

Comparing the equalities (23) and (24), we find

$$\sum_{j=1}^{n} j\alpha_j t^{j-1} = -\frac{p}{2}(p^2+q^2)(4\mu-\lambda^2) - p\gamma(t)u(0,0,t). \tag{25}$$

Now from (21) with $x=0$, $y=0$ we get

$$u(0,0,t) = \pm\frac{\sqrt{6(p^2+q^2)}i}{2}\sqrt{\lambda^2-4\mu}\frac{A\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi(0,0,t) + B\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi(0,0,t)}{A\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi(0,0,t) + B\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi(0,0,t)}. \tag{26}$$

Substituting (26) into (25) and taking into account (22) we obtain

$$\sum_{j=1}^{n} j\alpha_j t^{j-1} = -\frac{p}{2}(p^2+q^2)(4\mu-\lambda^2)\mp$$

$$\mp p\gamma(t)\frac{\sqrt{6(p^2+q^2)}i}{2}\sqrt{\lambda^2-4\mu}\frac{A\sinh(\frac{\sqrt{\lambda^2-4\mu}}{2}\sum_{j=1}^{n}\alpha_j t^j) + B\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\sum_{j=1}^{n}\alpha_j t^j}{A\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\sum_{j=1}^{n}\alpha_j t^j + B\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\sum_{j=1}^{n}\alpha_j t^j}. \tag{27}$$

The (27) is the algebraic system of equations for unknown α_j. Solving this algebraic system of equations we will define α_j as well as from (27) we define $u(0,0,t)$ then it is easy to find $\varphi(t)$ from (19). After that, from (21) we get soliton solutions of the loaded modified ZK equation.

When $\Delta = \lambda^2 - 4\mu > 0$, we get the following hyperbolic form of the solution of the loaded modified ZK equation

$$u(\xi) = \pm\frac{\sqrt{6(p^2+q^2)}i}{2}\sqrt{\lambda^2-4\mu}\frac{A\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + B\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}{A\cosh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + B\sinh\frac{\sqrt{\lambda^2-4\mu}}{2}\xi}, \tag{28}$$

When $\Delta = \lambda^2 - 4\mu < 0$, we get the following trigonometric form of the solution of the loaded modified ZK equation

$$u(\xi) = \mp\frac{\sqrt{6(p^2+q^2)}i}{2}\sqrt{4\mu-\lambda^2}\frac{A\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi + B\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}{A\cos\frac{\sqrt{4\mu-\lambda^2}}{2}\xi + B\sin\frac{\sqrt{4\mu-\lambda^2}}{2}\xi}. \tag{29}$$

Now, by choosing free parametrs we will write the travelling wave solutions of the loaded modified ZK equation in the simple form which can be used for the graphical illustrations.

1) when $(\lambda^2-4\mu) > 0$, $\lambda = 2\sqrt{3}$, $\mu = 2$, $p = \frac{1}{\sqrt{3}}$, $q = \frac{1}{\sqrt{3}}$, $A = 1$, $B = 0$ and $n = 1$, we have the following soliton solution

$$u(x,y,t) = 2ith(\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}y - t). \tag{30}$$

2) when $(\lambda^2-4\mu) < 0$, $\lambda = 0$, $\mu = 1$, $p = \frac{1}{\sqrt{3}}$, $q = \frac{1}{\sqrt{3}}$, $A = 1$, $B = 0$ and $n = 1$, we have the following periodical solution

$$u(x,y,t) = 2itg(\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}y - t). \tag{31}$$

4 Graphical representation of the loaded modified Zakharov-Kuznetsov equation

We have shown how to find the solutions of the loaded modified ZK equation in 3D plot formats to make it easier to imagine. Graphical representation is an effective tool for communication and it exemplifies evidently the solutions of the problems. The graphical illustrations of the solutions are depicted in the Figure 1 and Figure 2.

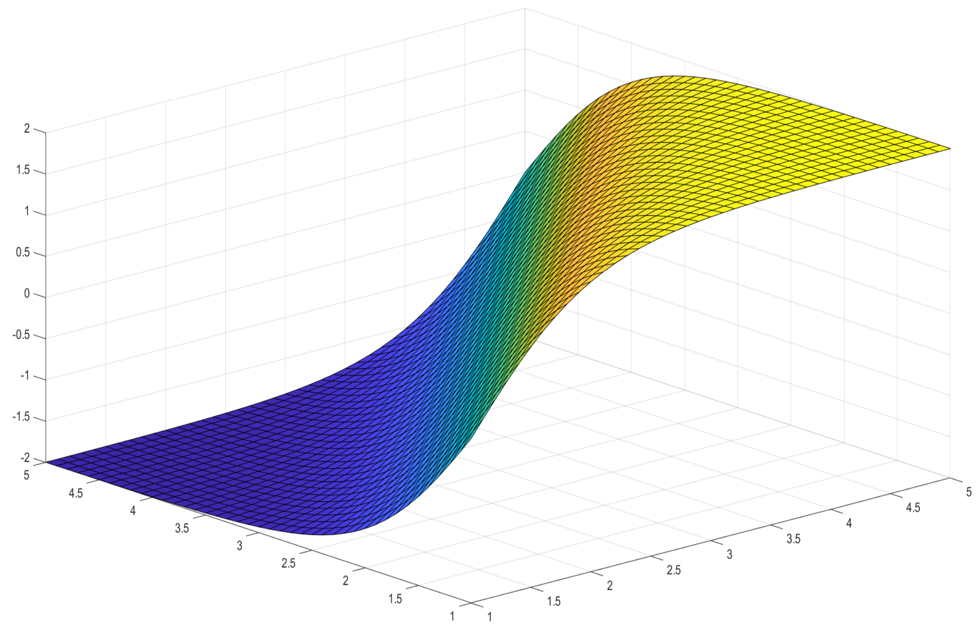

Figure 1: Imaginary part of the solution of the loaded modified Zakharov-Kuznetsov equation for $\lambda = 2\sqrt{3}$, $\mu = 2$, $(\lambda^2 - 4\mu) > 0$, $\lambda = 2\sqrt{3}$, $\mu = 2$, $p = \dfrac{1}{\sqrt{3}}$, $q = \dfrac{1}{\sqrt{3}}$, $A = 1$, $B = 0$, $y = 0$ and $n = 1$.

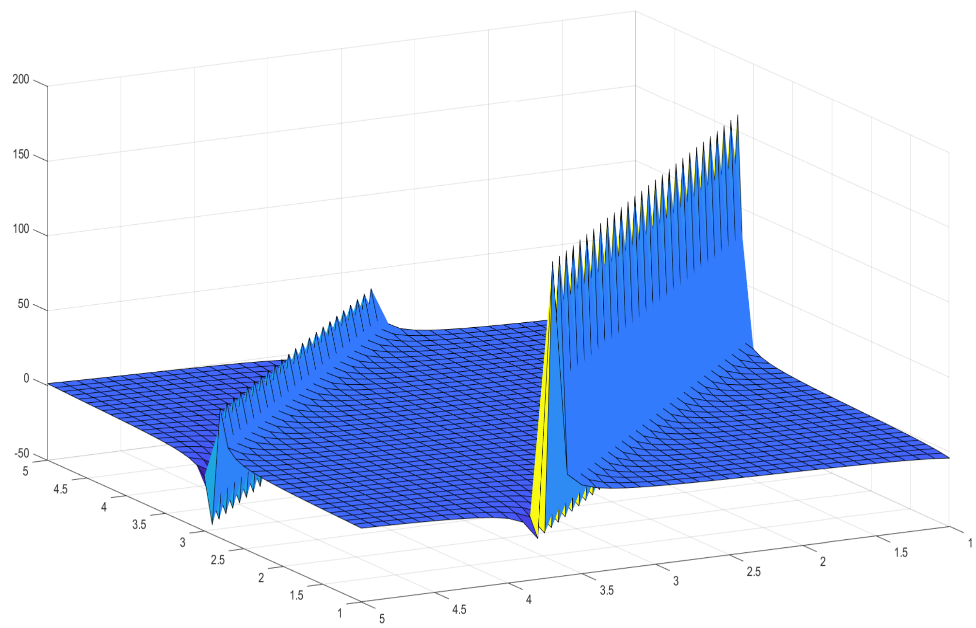

Figure 2: Imaginary part of the solution of the loaded modified Zakharov-Kuznetsov equation for $(\lambda^2 - 4\mu) < 0$, $\lambda = 0$, $\mu = 1$, $p = \dfrac{1}{\sqrt{3}}$, $q = \dfrac{1}{\sqrt{3}}$, $A = 1$, $B = 0$, $y = 0$ and $n = 1$.

5 Conclusion

In this article, we used the (G'/G) - expansion method to obtain the exact traveling wave solutions of the generalized forms of the loaded modified ZK equation. We have successfully obtained abundant traveling wave solutions. The solution procedure can be easily implemented in Matlab program. The graphical illustrations noticeably show the solitary solutions. The received solutions with free parameters may be important to explain some physical phenomena. It is shown that the performance of this method is productive, effective and well-built mathematical tool for solving nonlinear evolution equations.

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