Journal of Modern Technology and Engineering
Special Issue | IECHCI2022, 2023, pp.43-48

VISUALIZATION OF CONSTRUCTIVE FRACTAL OBJECT DESCRIPTIONS

Bahram Azizov, Asif Pashayev*

Azerbaijan University, Baku, Azerbaijan

Abstract. Modeling a natural phenomenon or process involves the use of its geometric structure. The fractal geometry unifies the geometric representation of reality and provides a universal mathematical apparatus for taking into account the influence of scales on a natural phenomenon or process. The paper shows constructive fractal objects that are built according to a given procedure using combinations of linear affine transformations of one and the same basic fragment.

Keywords: Fractal, affine transformations, imitation, modeling, Monte Carlo method, Random quantity, Statistical Testing method, probability.

***Corresponding author:** Asif Pashayev, Azerbaijan University, Baku, Azerbaijan,
e-mail: *asif.pashayev@au.edu.az*

Received: 9 October 2022; Revised: 18 November 2022; Accepted: 12 December 2022;
Published: 13 January 2023.

1 Introduction

Modern physical science is built on the consideration of many different latent and manifest symmetries that reflect the fundamental properties of space-time. Symmetries (invariance) are manifested in the form of preservation of the laws of change of certain properties of a material object, regardless of place and time. Hidden symmetries consist of the equality of particles of the same type, the existence of global constants, and many other manifestations.

Symmetry itself is rotational (for example, a circle coincides with itself as a result of rotation through any angle) and mirror reflection (there is no difference between right and left). The joint realization of mirror image, parallel translation, rotation and other invariance creates the beauty and perfection of natural objects. In the case of symmetry, the most important feature is that the laws of nature remain invariant during scale changes (Kronover, 2000). It creates clusters similar to itself (fractals: multiplying one and an object of the same composition by a certain number) or affine to itself (multifractals: a full spectrum of scales - a hierarchy of structures and processes) objects: crystals and neurons, snowflakes and lightning, etc (Hidayat et al., 2019).

Modeling of natural phenomena or process involves the use of its geometric structure. The emergence of fractal geometry creates a geometric representation of reality and provides a universal mathematical apparatus for considering the effect of scale in natural phenomena or processes. The widespread use of fractals and multifractals shows that fractal sets are unique and can be used in various fields of scientific knowledge. Scientists, researchers specializing in physics and other fields of science, as well as art, the great similarity of fractal figures with natural forms and the interdisciplinary feature of fractal geometry become an actual research field and increase interest in this part of modern geometry (Mandelbrot, 2002; Wang et al., 2022).

Constructive fractals are those geometric figures whose construction of a given combination procedure is obtained using combinations of linear (affine) transformations of the same basic fragment (parallel translation, rotation and expansion - compression).

Fractals have the following properties:

- always contains subsets of different sizes and with the same similarity coefficient.
- the main thing is that they cannot be written in a standard geometric language
- the size of a fractal figure is always larger than its topological size.
- often a fractal set is displayed as a combination of defined transformations.
- there are fractal sets (multifractals) with a dimension spectrum (combination of fractal subsets) (Katiyar et al., 2017; Al-Rawi, 2018).

Constructive fractals are given based on those transformations as a result of shrinking or growing based on a given procedure based on a given figure at the beginning.

The following linear transformations are used to construct constructive fractals.

$$\begin{cases} x' = x\cos\alpha - y\sin\alpha + x_0 \\ y' = x\sin\alpha + y_0 \end{cases} \tag{1}$$

These transformations define parallel translation and rotation.

If a parallelogram is taken as the base figure, the new parallelogram obtained after the transformation will retain its appearance and area (Fig.1).

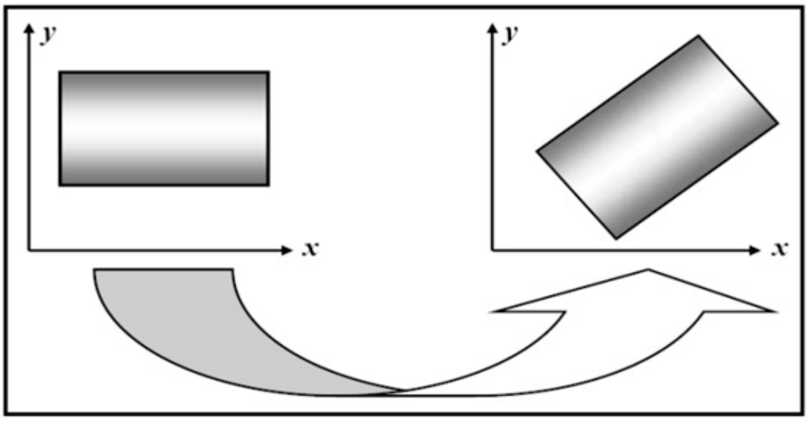


Figure 1: The new parallelogram obtained after the transformation

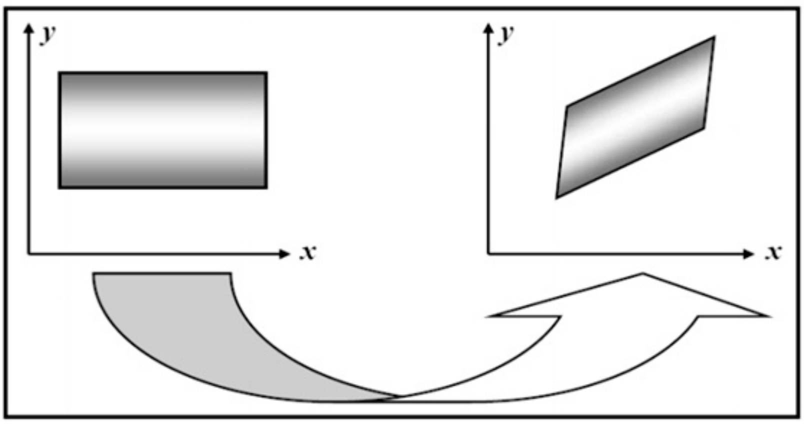


Figure 2: Affine transformation of a parallelogram

The parallel translation and rotation of a parallelogram is given as the following system equation.

$$\begin{cases} x' = ax + by + x_0 \\ y' = cx + dy + y_0 \end{cases} \tag{2}$$

This equation includes compression and expansion as well as translation and rotation. Affine transformation of a parallelogram.

(1) Fractal image obtained as a result of transformations Fig 3.

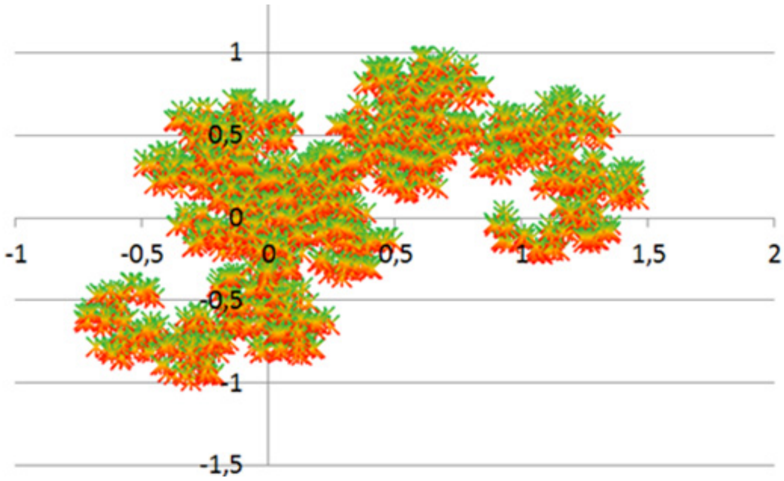


Figure 3: Affine fractal. Harter-Heighway dragon

Let's talk about affine transformations. Transformations are affine if they are mutually one-valued and the image of any straight line is a straight line. In order for the conversion to be mutually valuable, those different points should be viewed at different points. Any point goes to any point.

Conversion is self mapping of the set. If different elements pass into different elements, then such correspondence is called a bijective mapping.

In the special case, affine transformations simply provide motion (without any compression or dilation). Motion consists of parallel translation, rotation, various symmetries and their composition (Tregubova et al., 2018). One of the main properties of affine transformations is compression and expansion with respect to a straight line and a point. These are called similarity transformations or homotheties. The above geometric transformations are known from a classical geometry course. Regarding fractal sets, first of all it is necessary to show which geometric properties are preserved after affine transformations and what can be achieved by applying affine transformations. Affine transformations preserve the property of separating straight lines and two points.

$$A \neq B,\;\; f \in Aff \;\Rightarrow\; f(A) \neq f(B),$$

l is a straight line, $f \in Aff \;\Rightarrow\; f(l)$ is a straight line. Here Aff is the affine transformation of the original set. Two properties can be given in this way.

1.

$$\begin{matrix} \bullet A \\ \bullet B \end{matrix} \;\; \overset{f_{aff}}{\longleftrightarrow} \;\; \begin{matrix} \bullet A' \\ \bullet B' \end{matrix}$$

2.

$$/\, l \overset{f_{aff}}{\longleftrightarrow} l' /$$

3. The product of two affine transformations is also an affine transformation,

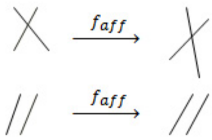
$$f, g \in Aff \;\Rightarrow\; (f \circ g) \in Aff$$

4. The inverse of an affine transformation is also an affine transformation

$$f \in Aff \;\Rightarrow\; f^{-1} \in Aff$$

5. Intersecting straight lines become intersecting straight lines in the affine transformation.
6. Parallel straight lines become parallel straight lines in the affine transformation.

We can show these features in the following way.



7. A parallelogram becomes a parallelogram as a result of an affine transformation.
8. A trapezoid turns into a trapezoid as a result of an affine transformation. The specified properties can be shown as follows.

9. One of the main properties of affine transformation is the conservation of fields. Let's look at the Affine transformation, which transfers two figures on a plane to one another. We can write the given properties formally.

$$f \in Aff,\;\; F_1' = f(F_1), \quad F_2' = f(F_2) \Rightarrow \frac{S_{F_1}}{S_{F_2}} = \frac{S_{F_1'}}{S_{F_2'}},$$

F_1' and F_2' are the images of figures F_1 and F_2 after several affine transformations, respectively. $S_{F_1}, S_{F_2}, S_{F_1'}, S_{F_2'}$ are their domains, respectively.

10. The length of straight line segments is preserved during affine transformations.

The last two properties are more important for constructive fractals. In practice, calculations in the construction of fractal sets depend on the areas and the lengths of the pieces. The initial fractal set can be obtained by applying geometric elements of affine transformations (Terekhov, 2011).

Finally, it should be noted that affine transformations play a major role in the representation of fractal sets in practice. The point is that the fractal clusters, which appear as real objects, actually consist of geometrical elements, and are geometrically irregular, complexly formed clusters.

Affine transformations help to form a fractal set, convenient for mathematical operations, moreover, properties 9 and 10 of affine transformations are quite adequate in this work.

Rotation, compression, stretching and inversion transformations are a special case of affine transformations of the plane and are given by the following formulas

$$\begin{cases} x_{n+1} = ax_n + by_n + e \\ y_{n+1} = cx_n + dy_n + f \end{cases} \tag{3}$$

$\sqrt{ad - cd}$ is the scaling factor.

The use of four viewing systems (3) (Iterative function system - IFS) allowed M. Barnsley to create fractal clusters very similar to natural structures.

Statistical tests using the Monte Carlo method are considered the simplest simulation modeling method in the absence of complete rules of behavior (Azizov & Panahova, 2021).

One of the most widespread methods of simulation modeling is the possibility of solving physical and mathematical problems of various types in the environment of the MS Excel application package using the Monte-Carlo or statistical tests method.

2 Construction of algebraic fractals using the Monte-Carlo method (method of statistical tests) in the MS Excel application software package environment

To describe algebraic fractals in MS Excel, it is necessary to use random number data according to the formula (RANDBETWEEN (0;1)) by choosing one of two systems of equations (Table 1).

Table 1. Choosing the system of equations

P is given by the formula **(RANDBETWEEN (0;1))**	
$if\ P = 0$	$if\ P = 1$
$\begin{cases} x_{n+1} = ax_n - by_n \\ y_{n+1} = bx_n + ay_n \end{cases}$	$\begin{cases} x_{n+1} = cx_n - dy_n + 1 - c \\ y_{n+1} = dx_n + cy_n - d \end{cases}$

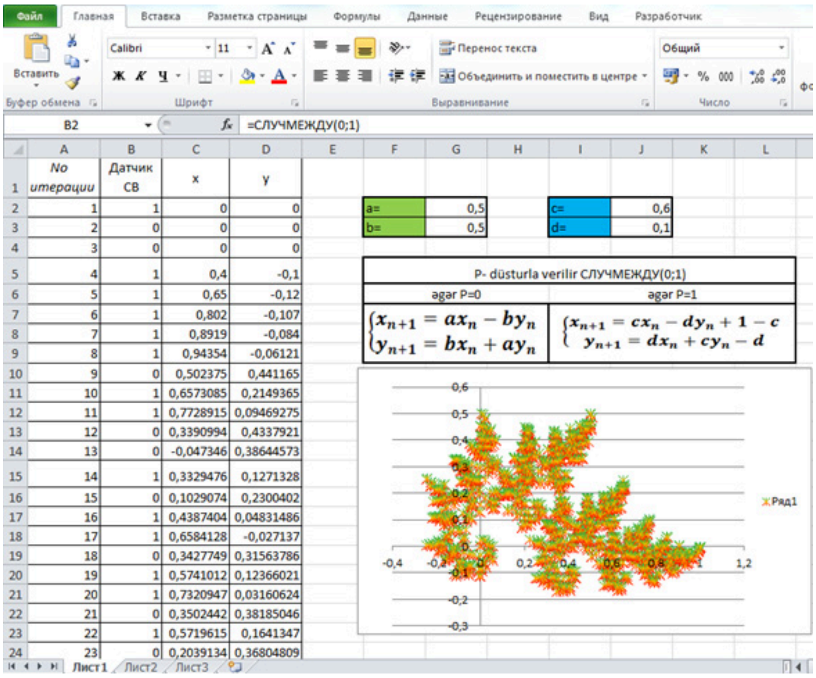


Figure 4: Construction of algebraic fractals using the Monte-Carlo method in the MS Excel application software package

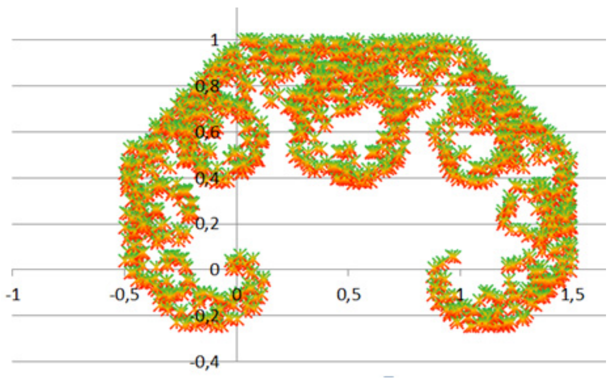


Figure 5: Levy C Curve ($a = 0,5; b = 0,5; c = 0,5; d = -0,5$)

References

Al-Rawi, A.T.H. (2018). Generation of the plants as a self-similarity fractal in three dimension by using lindenmayer system. *Revista AUS, 26*(2), 88-94.

Azizov, B.B., Panahova, Z.A. (2021). Theoretical study of algebraic affine transformations in solving geometrical problems in mathematics in the Geogebra software environment and analysis of application possibilities. *News of Pedagogical University. Mathematics and natural sciences series*, *69*(1), 30-47.

Hidayat, E.W., Putra, I.K.G.D., Giriantari, I.A.D., & Sudarma, M. (2019, July). Visualization of a two-dimensional tree modeling using fractal based on L-system. In I*OP Conference Series: Materials Science and Engineering* (Vol. 550, No. 1, p. 012027).

Katiyar, S. K., Reddy, K.M., & Chand, A.K.B. (2017, January). Constrained data visualization using rational bi-cubic fractal functions. In *International Conference on Mathematics and Computing* (pp. 265-277). Springer.

Kronover, R.M. (2000). *Fractals and Chaos in dynamic systems: Basics theory.* Moscow, Postmarket, 352.

Mandelbrot, B. (2002). *Fractal geometry of nature.* Moscow, Institute computer studies, 656.

Terekhov, S.V. (2011). *Fractals and physics of similarity.* Donetsk, Tsifrovaya typography, 255.

Tregubova, I.A., Sobko, K.O., & Gokhman, R.O. (2018). Fractal Graphics as the Modern Technology of Science. Digital Technology, 24, 105-111.

Wang, Y., Li, X., Wang, D., & Liu, S. (2022). A brief note on fractal dynamics of fractional Mandelbrot sets. *Applied Mathematics and Computation*, 432, 127353.