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AN IMPROVED BINARY INTEGER PROGRAMMING MODEL OF THE BAND COLLOCATION PROBLEM

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Abstract. In this paper, we consider a new recent combinatorial optimization problem called The Band Collocation Problem (BCP) which has a potential application in telecommunication networks. The BCP aims to design an optimal packing of information flows on different wavelengths into groups for obtaining the highest available cost reduction using wavelength division multiplexing technology. We improve its binary integer linear programming model. The improved model has been implemented in GAMS (the General Algebraic Modeling System) and solved using the CPLEX solver. We then create new problem instances with known optimal solutions to extend the current BCP Library for researchers to develop efficient computational solution methods.

Keywords: *Bandpass Problem, Band Collocation Problem, mathematical modeling, telecommunication, binary integer linear programming model.*

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1. Introduction

The Bandpass Problem (BP) whose first mathematical model was presented in 2009 is a combinatorial optimization problem which may be used in telecommunication systems [1]. Due to the development in the technology, some problems become invalid or useless and they need to be updated. In this sense, Nuriyev et. al. in 2015 announced the Band Collocation Problem (BCP) with its combinatorial mathematical model by extending the BP due to incompatibility with real life implementations at the present time [8]. Nuriyev et. al. also improved a nonlinear programming model for the BCP and introduced the Band Collocation Problem Library (BCPLib) which is meant to provide researchers with a set of test problems having various properties [9, 12]. Then, Gursoy et. al. presented a binary integer linear programming model for the BCP [4]. Kutucu et. al. gave a review of the mathematical formulations of the BCP and improved a simulated annealing meta-heuristic algorithm to solve the BCP [7]. Gursoy et. al. also presented the first heuristic algorithm to solve the BCP [5]. Afterwards, Nuriyeva developed a dynamic programming algorithm to find the minimum cost of a given traffic matrix that is a subproblem of the BCP [10]. Finally, Tekin, Keserlioglu and Gursoy presented a mathematical model for the BCP with limited resources [11]. In order to understand the BCP well, we first introduce the BP. The BP is related to transmitting data over fiber optic networks using the Dense

Wavelength Division Multiplexing (DWDM) technology [1]. The data is transmitted from a source to other stations on different wavelengths in a single fiber optic cable. Stations add/drop data onto/from the cable via an optical device called Add/Drop Multiplexers (ADM). Special cards in ADMs control each wavelength. They can add/drop(extract) data at some wavelengths to/from a network path [6]. Stations do not have to receive all data on the cable. We can see a simple illustration of a typical fiber optic network in Figure 1.

In Figure 1, there are one source, five stations, seven wavelengths (red, cyan, pink, yellow, blue, green, orange). We can see the special cards in the rectangular boxes indicated by colors, for example, red, pink, green and orange cards in the first station. It is understood that data carried on the first, third, sixth and seventh wavelengths have been requested by the first station. Thus, there are four special cards in the ADM of the first station for the current configuration. Each special card is responsible for one wavelength. However, according to the BP, a practical programmable ADM can add/drop multiple wavelengths if they are neighboring to each other, that is, if they are consecutive. In the BP, a group of consecutive wavelengths is called bandpass and the length of a bandpass is represented by a positive integer *B* called bandpass number.

Companies want to reduce the costs of constructing the network. Actually, this is the goal of the BP by maximizing the number of bandpasses. The key idea of the BP is to gather up requested data wavelengths at any station.

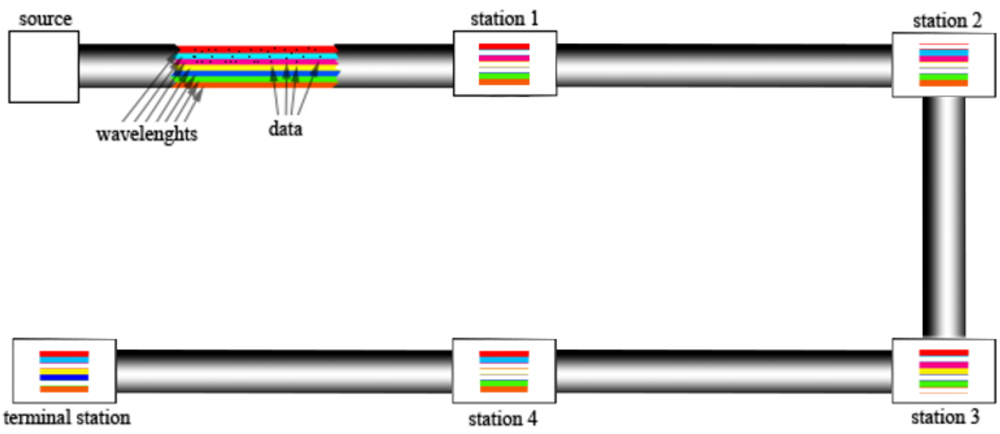

Figure 1. A simple illustration for a sample fiber optic network using DWDM.

Consider a communication network. The communication is conducted on m different wavelengths to carry data to be sent to n different stations. This situation is described by a binary matrix $A = a_{ij}$: if data carried on wavelength $i = 1,...,m$ is requested by station $j = 1,...,n$, then $a_{ij} = 1$, otherwise $a_{ij} = 0$. In order to represent the communication in Figure 1, we use a binary matrix A, with m = 7 and n = 5, corresponding to the network traffic shown in Figure 2(a) Each wavelength used to transmit data is colored in different colors. White color means that there is no any data transmitted to the sink station through the corresponding wavelength.

For this example, if each special card in ADMs controls one wavelength, then we need 21 cards. If some special cards are programmed to handle two consecutive wavelengths (this means that the bandpass number *B* is 2), then we would need 13 cards (see Figure 3(a), 8 for bandpass, 5 for single). Figure 3(b) shows a reordering of the rows of the matrix given in Figure 3(a). For this reordering, if some special cards are programmed for *B* = 2 consecutive wavelengths, then we need 11 cards (10 for bandpasses, 1 for single).

Special cards in ADMs are expensive and IT managers try to reduce the number of these cards used in ADM. How is it possible? The answer is to reorder the wavelengths. For this purpose, the BP asks reordering of the rows of a given matrix so that the number of *B* non-overlapping consecutive 1's is maximized. Note that in the communication network, reordering of the rows of the matrix simply corresponds to a reassignment of the DWDM wavelengths.

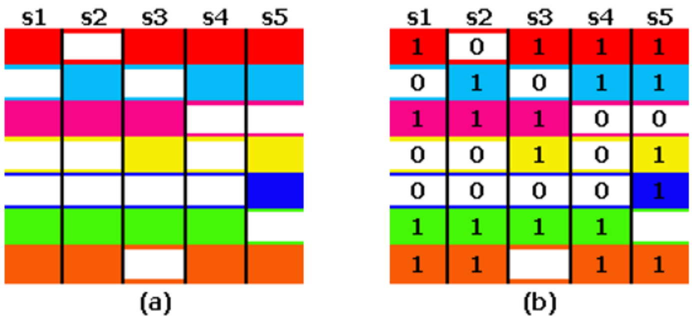

Figure 2. Optical network using 7 wavelengths (colors), (a) The network traffic in Figure 1, (b) The binary matrix of the network traffic in Figure 1.

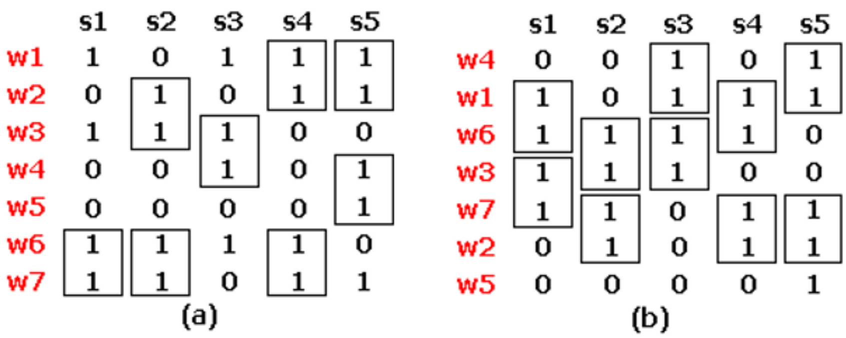

Figure 3. The Bandpasses when $B=2$ (a) Initial matrix having 8 bandpasses, (b) An optimal relocation of rows of the matrix having 10 bandpasses.

If some special cards are programmed for three consecutive wavelengths (this means $B=3$), then we would need 21 cards since there is no any bandpass when $B=3$ as it can be seen in Figure 4(a). However, Figure 4(b) shows optimal reordering of rows allows the use of fewer cards when $B=3$. then we need 11 cards (5 for bandpasses, 6 for single).

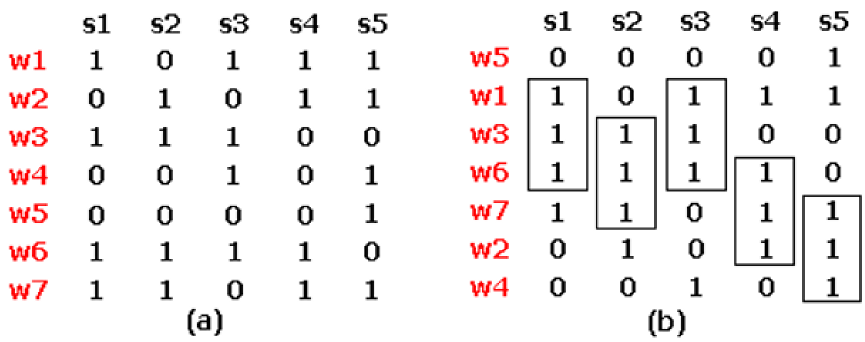

Figure 4. The bandpasses when $B=3$ (a) Initial matrix having no bandpasses (b) An optimal relocation of rows of the matrix with five bandpasses.

It is clear that to find an optimal solution to the BP, we must perform an exhaustive search over all row permutations. There are total of *m!* different permutations of m wavelengths. This number grows faster than exponentially with *m*. Therefore, this is not reasonable. Babayev et al. also have proved that the BP is NP-complete [1]. Recent changes in ADM technology made the BP ineffective. We can explain the deficiencies of the BP as follows:

- Technology allows an ADM to drop a wavelength even if it doesn't carry any information. Therefore, a bandpass may contain zero elements.
- The bandpasses may be in different sizes. That is, *B* is not fixed.
- The BP ignores costs of the programmable cards.

From all reasons above, we need a new problem named as the Band Collocation Problem. In the next section, we introduce the Band Collocation Problem (BCP) and present its improved binary integer linear model. Furthermore, in Section 3, we create new problem instances with known optimal solutions to extend the current BCP Library for researchers to develop efficient computational solution methods.

2. The Band Collocation Problem and its binary integer linear programming model

Let $A=(a_{ij})$ be a binary matrix of dimension mxn, $B_k=2^k$ be the length of a B_k-band and c_k be the cost of a B_k-band, where $k=1,...,\mathrm{t}=\lfloor \log_2 m \rfloor$. A B_k-band can include less than 2^k consecutive 1's, that is, it may include zero entries. The BCP is consists of finding an optimal permutation of rows of the matrix that minimizes the total cost of B_k-bands in all columns.

In Figure 5, we consider the matrix of Figure 2 as an instance of the BCP with the band costs c_0 = 1000, c_1 = 1950 and c_2 = 3810. The band collocation cost of the matrix in Figure 5(a) is 20600 with five B_0-band (green boxes) and eight B_1-band (blue boxes). After a relocation of row as in Figure 5(b), the band collocation cost is 20140 with one B_0-band (green box), two B_1-band (blue boxes) and four B_2-band (yellow boxes).

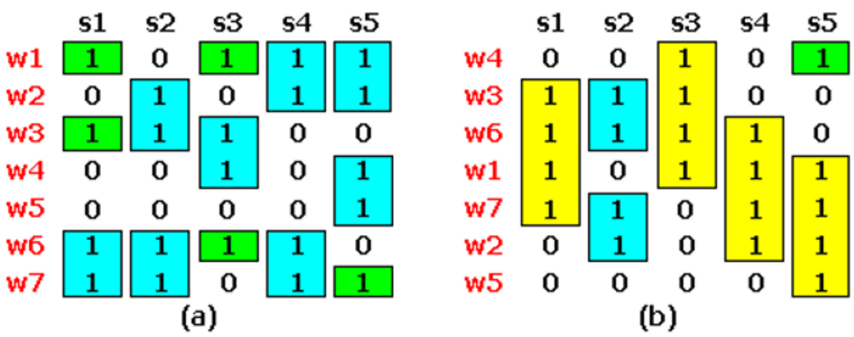

Figure 5. A BCP instance, (a) The binary traffic matrix in Figure 2(b) with bands, (b) An optimal relocation of rows of the matrix and optimal band locations.

We define the decision variables as follows:

$$x_{ir} = \begin{cases} 1, & \text{if row } i \text{ is relocated to position } r \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ij}^{k} = \begin{cases} 1, & \text{if row } i \text{ is the first row of a } B_k\text{-band in column } j \\ 0, & \text{otherwise} \end{cases}$$

$$z_{ij}^{k} = \begin{cases} 1, & \text{if } a_{ij} \text{ is an element of a } B_k\text{-band in column } j \\ 0, & \text{otherwise} \end{cases}$$

where $i, r = 1,...,m$, $j = 1,...,n$ and $k = 1,...,t$. We improve the binary integer linear programming model of the BCP as follows:

$$Minimize \sum_{k=0}^{t} \sum_{i=1}^{m-2^k+1} \sum_{j=1}^{n} c_k y_{ij}^{k} \tag{1}$$

subject to

$$\sum_{r=1}^{m} x_{ir} = 1 \qquad i = 1,...,m \tag{2}$$

$$\sum_{i=1}^{m} x_{ir} = 1 \qquad r = 1,...,m \tag{3}$$

$$2^k y_{lj}^{k} \le \sum_{r=l}^{l+2^k-1} \sum_{i=1}^{m} z_{rj}^{k} \qquad k = 0,...,t,\ j = 1,...,n,\ l = 1,...,m-2^k+1 \tag{4}$$

$$\sum_{k=0}^{t} \sum_{i=1}^{m-2^k+1} 2^k y_{ij}^{k} \ge \sum_{i=1}^{m} a_{ij} \qquad j = 1,...,n \tag{5}$$

$$\sum_{i=1}^{m-2^k+1} 2^k y_{ij}^k = \sum_{i=1}^{m} z_{ij}^k \qquad k=0,...,t\,,\ j=1,...,n \tag{6}$$

$$\sum_{i=l}^{l+2^k-1} y_{ij}^k \le 1 \qquad k=0,...,t\,,\ j=1,...,n\,,\ l=1,...,m-2^k+1 \tag{7}$$

$$\sum_{k=0}^{t} z_{ij}^k \ge \sum_{r=1}^{m} a_{rj}\cdot x_{ri} \qquad i=1,...,m\,,\ j=1,...,n \tag{8}$$

$$x_{ir} \in \{0,1\},\, y_{ij}^k \in \{0,1\},\, z_{ij}^k \in \{0,1\} \qquad i,r=1,...,m\,,\ j=1,...,n\,,\ k=0,...,t \tag{9}$$

Constraints (2) express the fact that row i must be relocated into one new position r only, constraints (3) express that only one row i must be relocated to each new position r. Constraints (4) guarantee to find the coordinates of B_k-bands, (5) say that the total length of all B_k-bands in column *j* cannot be less than the number of 1's in the same column. Constraints (6) ensure that the number of entries included in a B_k-band is exactly the length of that B_k-band. Constraints (7) guarantee that any entry of the matrix is the first entry of a unique B_k-band. Constraints (8) say that each non-zero entry of the resulting matrix has to be an element of a B_k-band.

The aim is to find an optimal relocation of rows of the matrix that minimizes the total cost of all B_k-bands (1) subject to the constraints (2)-(9).

The previous model of the BCP has nine set of functional constraints other than integral constraints. However, in this new model, there are seven functional constraints.

3. Online library

The BCP which is an extension of the Bandpass problem is also NP-hard. Therefore, heuristic or metaheuristic algorithms are needed to be improved for solving the BCP. Besides, researchers working on this problem will need problem instances (matrices) to check and compare their solutions, running times for the algorithms or iterations for the mathematical models.

The Band Collocation Problem Library (BCPLib) which is available online at http://fen.ege.edu.tr/~arifgursoy/bps/ [12] is introduced by Nuriyev et. al. [9, 7]. The first version of the BCPLib includes only instances with unknown optimal values. The matrices are created according to the following parameters:

- Number of rows (m),
- Number of columns (n),
- Cost of a B_k-Bank (c_k),
- Density of the binary matrix.

In the library, there are three different matrices (TX-M1, TX-M2 and TX-M3) of the same type (the same number of rows and columns) and six different cost alternatives for each matrix. The cost alternative is determined by the growth rate (ρ = 0.05, 0.1, 0.2, 0.3, 0.4 and 0.5). The cost of each B_k-band is calculated by the following formula $c_{k+1} = (2-\rho)c_k$, where $k = 0,1,...,t = \lfloor \log_2 m \rfloor$. The density of a matrix is the ratio of the number of its nonzero entries to the total number of its entries. The densities are 35%, 50% and 75%. The BCPLib has 72 matrix types, and so 216 different matrices and totally 1296 problem instances.

It is obvious that if all 1's in each column are consecutive, then the total cost is minimum regardless of the matrix type, the density and the increasing rate. Therefore, in order to create problem instances with known optimal cost values, we constructed 1296 problem instances according to the parameters mentioned above. These new instances named from OT1 to OT72 are published in the BCPLib [12].

4. Computational experiments

In this section, we present numerical results obtained for the BCP. The BIP model has been implemented in GAMS [3] and solved by the CPLEX solver in NEOS server [2]. The test problems with known optimal values are taken from the BCPLib.

The computational results are given in Table 1. In Table 1, ratio is the growth rate of the costs, m is the number of rows, n is the number of columns, d is the density of non-zero elements of the matrix in percentage, Optimal is the optimal value of the problem instance (matrix), BIP Solution is the value obtained using the BIP model (1)-(9), Gap is the relative error (in percentage) between the optimal value and the BIP solution, Time is the CPU time in seconds.

We can see in Table 1 that the BIP model has solved to optimality 26 instances out of 32 and CPU time varies from 0.17 second to 8 hours.

Table 1. Computational results using the BIP model in NEOS solver.

Instance	ratio	m	n	d %	Optimal	BIP Solution	Gap %	Time (sec)
OT1M3	0.10	12	6	35	23140	23140	0	0,22
OT1M3	0.50	12	6	35	15880	15880	0	0,37
OT4M3	0.10	16	8	35	40180	40180	0	0,64
OT4M3	0.50	16	8	35	22520	22520	0	0,58
OT7M3	0.10	24	10	35	73670	73670	0	32,24
OT7M3	0.50	24	10	35	39050	39050	0	2,56
OT10M3	0.10	32	12	35	115690	115690	0	236,77
OT10M3	0.50	32	12	35	55860	55860	0	33,93
OT16M3	0.10	48	16	35	226930	226930	0	1566,58
OT16M3	0.50	48	16	35	97520	97520	0	131,93
OT22M3	0.10	64	20	35	370670	370670	0	9507,17

OT22M3	0.50	64	20	35	142400	142400	0	3634,77
OT31M3	0.10	88	26	35	641820	774910	17,2	28700.06
OT31M3	0.50	88	26	35	219610	219610	0	27137,42
OT70M3	0.10	96	16	35	432390	450820	4,1	28795,26
OT70M3	0.50	96	16	35	142330	142330	0	19246.34
OT2M3	0.10	12	6	50	33160	33160	0	0,88
OT2M3	0.50	12	6	50	20150	20150	0	0,17
OT5M3	0.10	16	8	50	56080	56080	0	1,22
OT5M3	0.50	16	8	50	30540	30540	0	0,83
OT8M3	0.10	24	10	50	103190	103190	0	12,14
OT8M3	0.50	24	10	50	46940	46940	0	0,77
OT11M3	0.10	32	12	50	161150	161150	0	209,05
OT11M3	0.50	32	12	50	66640	66640	0	39,00
OT17M3	0.10	48	16	50	313990	313990	0	214,27
OT17M3	0.50	48	16	50	116190	116190	0	161,83
OT23M3	0.10	64	20	50	523370	526510	0,6	28000,08
OT23M3	0.50	64	20	50	178000	178000	0	506,43
OT32M3	0.10	88	26	50	905380	1061260	14,7	28000,17
OT32M3	0.50	88	26	50	279130	280630	0,5	28000,06
OT71M3	0.10	96	16	50	605150	638560	5,2	28700.04
OT71M3	0.50	96	16	50	182940	182940	0	4460.01

5. Conclusion

In this paper, we developed a new binary integer linear programming model for the Band Collocation Problem. The model has been implemented in GAMS and solved using the CPLEX solver in NEOS server. We present some computational results for the problem. We then improved the current Band Collocation Problem Library (BCPLib) which is meant to provide researchers with a set of test problems with known optimal solutions. Developing metaheuristic algorithms is the issue of the ongoing research.

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