

## MATHEMATICAL MODELS OF MULTI-COORDINATE ELECTROMECHATRONIC SYSTEMS OF INTELLECTUAL ROBOTS

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**Abstract.** In the article mathematic description of mechatronic module in intelligent robots and robotic systems is observed. To display the structure of mechatronic modules, which consist of interrelated different electrical, magnetic and mechatron components, mathematic models by the help of multiplication theory based on relations and visualising lines and points are designed. Analytical explanation of mechatronicmodule in intelligent robots is provided.

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**Keywords:** robotic systems, mechatronicmodule, mathematic models, intelligent robots.

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## 1 Introduction

The mathematical description of multicoordinate mechatronic modules (MMM) is the most important stage of the theoretical analysis and synthesis of modules of intelligent robots and robotic systems. At present, the absence of models suitable for a detailed description of the structures of such systems precludes the possibility of automating the design of specific structures (Gomilko et al., 2016).

An element of MMM can be identified with a system that has the properties of material reproducibility and repeatability; therefore, it is naturally possible to define a formal theory of MMM and its model in the language of set theory, as is customary in systems theory (Nazarov, 2005).

To display the structures of MMM, consisting of interconnected heterogeneous electrical, magnetic and mechanical components, in the work there are set-theoretic models operating with the following concepts: relations, mappings, unified relation, lines and points:

$$S = \langle L, P, R_L, \pi, R_{L\pi} \rangle,$$

$$S^{-1} = \langle P, L, R_L, \pi, R_{\pi L} \rangle,$$

where  $L, P$  are the many lines and points;  $R_L$  is an equivalence relation given in the set  $L$ ;  $\pi$  is a combined relation given in the set  $P$ ;  $R_{L\pi}$  is a mapping of the set  $L$  in the set  $\pi$ .

Combined relations in the work are called the combination of binary and ternary relations.

S, S-1 models are transformed into other forms depending on the properties of the relations

RL,  $\pi$  of  $RL\pi$  mappings and the goals of the problem:

$$\begin{aligned} S_1 &= \langle L, P, \mathcal{L}, P, \Gamma \rangle; \\ S^{-1} &= \langle P, L, P, \mathcal{L}, \Gamma^{-1} \rangle; \\ T &= \langle L, P, P, C, \Gamma \rangle; \\ T^{-1} &= \langle P, L, P, C^{-1}, \Gamma^{-1} \rangle, \end{aligned}$$

where  $\mathcal{L}, P$  are families of classes of lines and points, respectively,

$$L_i \in L, P_i \in P \text{ for } i = \overline{1, n}, j = \overline{1, m},$$

$C$  is a family of properties.  $C^1$  is a reverse family of properties;  $C_i$  are properties,

$$C_i = \left\{ \left\{ C_{iki}^j \right\} \right\}, k = \overline{1, l_{ki}}, j = \overline{1, P_{ki}};$$

$C_{iki}^j$  are private properties,

$$C_{iki}^j = C_{iki}^0 \cap P_i;$$

$C_{iki}^0$  is an original line property  $l_{iki}$ .

$\Gamma, \Gamma^{-1}$  is a structural and inverse structural sets, respectively;

$\Gamma = \{\Gamma_i\}, i = \overline{1, l}; \Gamma_i$  is a structural class,

$$\Gamma_i = \{\Gamma_{iri}\}, k = \overline{1, l},$$

$\Gamma_{iri}$  is a structural element;  $\Gamma_{iri} = \frac{\{C_{iki}\}}{l_{iki}}, j = \overline{1, P_{ki}}$ .

Three pairs of forms of models (S, S-1, S, S-1, T, T-1) are equivalent in the semantic content of the information presented by them about the system element and differ from each other only in the ways of representation. When operating with models, the interchangeability of components becomes important, which determines the possibility of excluding from consideration certain components of a model at various stages of its use. The latter becomes possible if the necessary part of information about the deleted components of the model is saved in the remaining ones. In models S, S1, T information about the set L is completely contained in the sets PL, RL $\pi$ ,  $\Gamma$ ,  $\mathcal{L}$ ; information about the set of P- in the sets  $\pi$  R $\pi$ L, G-1, P; information about RL is contained entirely in the sets  $\mathcal{L}, \Gamma$ , and partly in sets C. Information on RL $\pi$  is partially contained in the sets  $\Gamma$  and C. This is shown in Table 1 by graphs whose vertices indicate the amount of information about the components inscribed at the vertices. Solid arcs indicate the complete inclusion of information about a component located at the beginning of the arc. Dotted arcs indicate partial inclusion of information. From Table 1 it can be seen that the exclusion of the component G from the model S1, the components RL $\pi$  from the model of type T is impossible, since in this case the remaining parts of the models lose connectivity.

Comparison of models shows a great flexibility of the type T model, which keeps the remaining parts connected even if the component T is removed with the largest amount (number of incoming arcs) of information, which is impossible in other models (Satapathy et al., 2013). This property of the model allows, in the intermediate calculations, to temporarily exclude some of its components and easily transform the structural set  $\Gamma$ , the family of the line  $\mathcal{L}$  or the family of properties C by removing one of the floors in the expression of the structural element. Based on these considerations, in the work of the model T and T-1 taken as the basis and called structural models.

Analysis of the models showed that all information about the components L,  $\mathcal{L}, C$  (components P, P, C-1) of the structural model T (models T-1) is completely contained in the structural set G (structural inverse set G-1). This allows us to uniquely find the expression of the components L,  $\mathcal{L}, C$  (P, P, C-1) of the structural model by the expression of the structural (reverse structural) set and reduce the task of representing the structural model to the representation of the structural set.

№	Types of models	Model graphs
1	S $\langle K, P, R_L, \underline{II}, R_{LII} \rangle$	
2	S <sub>I</sub> $\langle L, P, \underline{L}, P, L \rangle$	
3	T $\langle L, P, \underline{L}, C, L \rangle$	

Definition the partition of the linear (point) property corresponding to the partition of  $\mathcal{L}$  (partition of P) is called the colored linear (point) property. Structural sets  $\Gamma$  can be linearly colored ( $\Gamma\Delta$ ), point-colored  $\Gamma_T$  and linearly point-colored ( $\Gamma\Delta$ ) depending on the partitions  $\xi$  and  $P$ . Linearly - pointwise (LT) colored MMM structural sets have the form:

$$\Gamma^{\Delta T} = \{ \Gamma_{\mathcal{L}}^{\Delta T}, \Gamma_M^{\Delta T}, \Gamma_{MX}^{\Delta T}, \Gamma_{B3}^{\Delta T} \},$$

where,  $\Gamma_{\mathcal{L}}^{\Delta T}, \Gamma_M^{\Delta T}, \Gamma_{MX}^{\Delta T}$  are the colored structures of the electrical, magnetic, and mechanical subsystems, respectively;  $\Gamma_{B3}^{\Delta T}$  are the colored structural sets of the relationship subsystem, which are represented as

$$\Gamma_{\mathcal{L}}^{\Delta T} = \{ \Gamma_{\mathcal{L}1}^{\Delta T}, \Gamma_{\mathcal{L}2}^{\Delta T}, \dots, \Gamma_{\mathcal{L}K}^{\Delta T}, \dots, \Gamma_{\mathcal{L}C}^{\Delta T} \},$$

$$\Gamma_M^{\Delta T} = \{ \Gamma_{M1}^{\Delta T}, \Gamma_{M2}^{\Delta T}, \dots, \Gamma_{MK}^{\Delta T}, \dots, \Gamma_{MC}^{\Delta T} \},$$

$$\Gamma_{MX}^{\Delta T} = \{ \Gamma_{MX1}^{\Delta T}, \Gamma_{MX2}^{\Delta T}, \dots, \Gamma_{MXK}^{\Delta T}, \dots, \Gamma_{MXC}^{\Delta T} \},$$

$$\Gamma_{B3}^{\Delta T} = \{ \Gamma_{l31}^{\Delta T}, \Gamma_{l32}^{\Delta T}, \dots, \Gamma_{l3K}^{\Delta T}, \dots, \Gamma_{l3C}^{\Delta T} \}.$$

$\Gamma_{\mathcal{L}K}^{\Delta T}, \Gamma_{MK}^{\Delta T}, \Gamma_{MXK}^{\Delta T}$  are the colored structural elements of the electrical, magnetic and mechanical subsystems, respectively;

The painted structural element of the relationship subsystem  $\Gamma_{l3K}^{\Delta T}$  is represented as

$$\Gamma_{\mathcal{L}K}^{\Delta T} = \frac{C_{\mathcal{L}K}}{l_{\mathcal{L}K}}, \Gamma_{MK}^{\Delta T} = \frac{C_{MK}}{l_{MK}}, \Gamma_{MXK}^{\Delta T} = \frac{C_{MXK}}{l_{MXK}}, \Gamma_{l3K}^{\Delta T} = \frac{C_{l3K}}{l_{l3K}}.$$

Here  $C_{\mathcal{L}K}, C_{MK}, C_{MXK}$  is a family of properties of the electrical, magnetic, and mechanical

subsystems, respectively.  $C_{l_{3K}}$  – family of properties of the relationship subsystem.

$$\begin{aligned} C_{\mathfrak{e}K} &= \left\{ C_{\mathfrak{e}K}^1, C_{\mathfrak{e}K}^2, \dots, C_{\mathfrak{e}K}^j, \dots, C_{\mathfrak{e}K}^{P_{\mathfrak{e}K}} \right\}, \\ C_{MK} &= \left\{ C_{MK}^1, C_{MK}^2, \dots, C_{MK}^j, \dots, C_{MK}^{P_{MK}} \right\}, \\ C_{MXK} &= \left\{ C_{MXK}^1, C_{MXK}^2, \dots, C_{MXK}^j, \dots, C_{MXK}^{P_{MXK}} \right\}, \\ C_{l_{3K}} &= \left\{ C_{l_{3K}}^1, C_{l_{3K}}^2, \dots, C_{l_{3K}}^j, \dots, C_{l_{3K}}^{P_{l_{3K}}} \right\}. \end{aligned}$$

Here  $C_{\mathfrak{e}K}$ ,  $C_{MK}$ ,  $C_{MXK}$  are the families of particular properties of the electrical, magnetic, and mechanical subsystems, respectively;  $C_{l_{3K}}$  is a family of private properties of the relationship subsystem:

$$\begin{aligned} C_{\mathfrak{e}K}^j &= \left\{ P_{\mathfrak{e}Kj}^1, P_{\mathfrak{e}Kj}^2, \dots, P_{\mathfrak{e}Kj}^l, \dots, P_{\mathfrak{e}K}^{P_{\mathfrak{e}Kj}} \right\}, \\ C_{MK}^j &= \left\{ P_{MKj}^1, P_{MKj}^2, \dots, P_{MKj}^l, \dots, P_{MK}^{P_{MKj}} \right\}, \\ C_{MXK}^j &= \left\{ P_{MXKj}^1, P_{MXKj}^2, \dots, P_{MXKj}^l, \dots, P_{MXK}^{P_{MXKj}} \right\}, \\ C_{l_{3K}}^j &= \left\{ P_{l_{3Kj}}^1, P_{l_{3Kj}}^2, \dots, P_{l_{3Kj}}^l, \dots, P_{l_{3K}}^{P_{l_{3Kj}}} \right\}. \end{aligned}$$

Here  $P_{\mathfrak{e}Kj}^l, P_{MKj}^l, P_{MXKj}^l$  are the colored sets of points of the electrical, magnetic and mechanical subsystems, respectively;

$P_{l_{3K}}^l$  is  $\Delta T$  colored set of interconnection subsystem points:

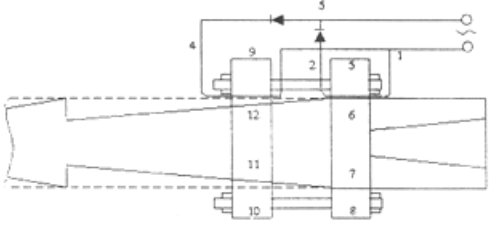
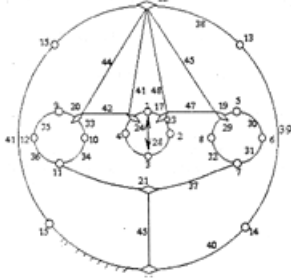
$$\begin{aligned} P_{\mathfrak{e}Kj}^l &= \{P_{\mathfrak{e}1}, P_{\mathfrak{e}2}, \dots, P_{\mathfrak{e}n}, \dots, P_{l_{\mathfrak{e}pl}}\}, \\ P_{MKj}^l &= \{P_{M1}, P_{M2}, \dots, P_{Mn}, \dots, P_{l_{MPl}}\}, \\ P_{MXKj}^l &= \{P_{MX1}, P_{MX2}, \dots, P_{MXn}, \dots, P_{l_{MXPl}}\}, \\ P_{l_{3K}}^l &= \{P_{b31}, P_{b32}, \dots, P_{b3n}, \dots, P_{l_{b3pl}}\}. \end{aligned}$$

Here  $P_{\mathfrak{e}n}, P_{Mn}, P_{MXn}$  are the points of electrical, magnetic and mechanical subsystems, respectively;  $P_{b3n}$  is the interconnection subsystem point;  $l_{\mathfrak{e}K}, l_{MK}, l_{MXK}$  are the lines of electrical, magnetic and mechanical subsystems, respectively;  $l_{b3K}$  is the interconnection subsystem line.

Table 2 shows the LD structural sets of a particular mechatronic module (Klimov, 1999) which are compiled according to its topological image. In this case, the topological image of MMM is constructed using the abstract concepts of "line" and "dot".

In contrast to the usual ways of displaying MMM graphs of schemes (for example, pole graphs) (Zimina et al., 2016), which do not allow to indicate the entire topology of interconnections, the latter are described in terms by graphs with two- and three-dimensional points. The MMM graph (Table 2) contains mentally divided four subgraphs: electrical, magnetic, mechanical, and interconnections. Winding on the magnetic core in the MMM column corresponds to the line (25) connecting one-dimensional points (1 and 2). At the same time. (No.1-4) - points of the electrical subsystem; (No. 5-12) - points of the magnetic subsystem; (No. 12-16) - points of the mechanical subsystem; (No. 17-18) - interconnection points (two-dimensional) of

the electrical subsystem; (No. 19-21) - points of interrelations of the magnetic subsystem; (No. 22-23) - interconnection points of the mechanical subsystem; (No. 24-28), (No. 29-36), (No. 37-41) - lines of the electrical, magnetic, mechanical subsystem; (No. 42-48) - interconnection subsystem lines.

<p><u>Mechatronic module</u></p>	<p>1</p>	
<p><u>Topological image</u></p>	<p>2</p>	
<p><u>Analytical description</u></p>	<p>3</p>	$\left[ \frac{1/4/D18}{24}, \frac{1/2/D17}{25}, \frac{2/3}{26}, \frac{3/4}{27}, \frac{1/8}{28} \right],$ $\left[ \frac{8/5/D19}{29}, \frac{5/6}{30}, \frac{6/1}{31}, \frac{1/8}{32}, \frac{9/10/D20}{33}, \frac{10/11}{34}, \frac{12/9}{36}, \frac{11/12}{35}, \frac{1/11/D21}{37} \right],$ $\left[ \frac{16/13/D22}{38}, \frac{13/14}{39}, \frac{14/15/D23}{40}, \frac{15/16}{41} \right],$ $\left[ \frac{D18/D20}{42}, \frac{D17/D19}{43}, \frac{D20/D22}{44}, \frac{D19/D22}{45}, \frac{D21/D23}{46}, \frac{D18/D22}{47}, \frac{D17/D22}{48} \right].$

The set-theoretic description of the structures of multi-axis mechatronic modules of intelligent robots and robotic systems allows displaying structures from interconnected physical heterogeneous systems and determining the set of possible structures of modules.

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