Journal of Modern Technology and Engineering
Vol.5, No.1, 2020, pp.85-96

A SURVEY OF MONTE CARLO DENOISING: CHALLENGES AND POSSIBLE SOLUTIONS

Sermet Mir*

Yasar University, Izmir, Turkey

Abstract. In the field of computer graphics, path tracing is a widely used technique for generating photorealistic images. The technique is based on Monte Carlo methods and random sampling is used in the solution of the rendering equation. Therefore, the quality of the output images is dependent on the number of samples used in the rendering operation. It may become noisy if a sufficient number of samples is not provided in the computation and it may converge slowly as the number of samples is increased. In order to overcome this problem, researchers came up with the idea of adaptive sampling and using a denoising filter as a post-processing technique. This paper analyzes state-of-the-art studies focusing on these subjects. It also examines the limitations and challenges that can be seen in such applications. Finally, some of the open-research areas for further investigation has been mentioned.

Keywords: Monte Carlo simulations, histograms, denoising, post-processing filters.
AMS Subject Classification: 54A20, 62D05, 68U05, 68U10, 65D18.
Corresponding author: Sermet Mir, Yasar University, Universite str., No.37-39, Agacli Yol - Bornova, Izmir, Turkey, Tel.: +902325708296, e-mail: sermet.onel@yasar.edu.tr
Received: 12 October 2019; *Accepted: 25 March 2020;* *Published: 30 April 2020.*

1 Introduction

Monte Carlo path tracing is a technique that offers a general solution for simulating the light behavior in the rendering process. In parallel to the improvements in the hardware and software technology and its nature of physically-based simulation of the sample scenes, the popularity of path tracing is increasing in the computer graphics based applications. However, the technique is based on Monte Carlo methods which employ random sampling in the solution of the rendering integral (Kajiya, 1986). Therefore, the quality of the output images depends on the number of samples used in the computation. In other words, using a low number of samples in the computation may lead to noisy pixels whereas an increase in the number of samples may lead to a slow computational convergence rate which creates performance problems. This bottleneck leads researchers to study alternative solutions. These studies can be classified in two groups: adaptive sampling and denoising filtering methods (Delbracio et al., 2014).

The adaptive sampling methods follow the idea of choosing locally adapted samples with the characteristics of the processed computation. Therefore, it becomes easier to compute the rendering integral with better positioned samples and new samples are added only when it is necessary in the computation. This approach minimizes the required number of samples which results in reduced execution time. Hence, we can say that adaptive sampling methods take action during the rendering operation. A number of studies follow this strategy to improve the quality of the path traced scenes (Soler et al., 2009; Egan et al., 2011b; Lehtinen et al., 2011; Herholz et al., 2016).

Denoising filtering methods can be classified as post-processing strategies. Basically, these methods use the information gathered during the rendering process and then apply a filtering procedure to remove noisy pixels on the image space. The simpler models use pixel color and neighborhood information where more complex models take sampling information into consideration (Delbracio et al., 2014; Rousselle et al., 2012). Moreover, it is possible to combine these strategies and a generalized model can be proposed using the advantages of both approaches. By tracking the sample information and combining it with the pixel color information, a more robust model can be built resulting in higher accuracy in the output image.

Furthermore, the emerging trend of using machine learning algorithms is also addressed in the denoising procedure. Using convolutional neural networks (CNN) for kernel prediction (Bako et al., 2017) or interactive auto-encoding (Chaitanya et al., 2017) are some of the examples that can be given in this group. Some studies also focus on deep compositing adding the depth information compared to flat images. The main motivation behind such approaches relies on the limitations of denoising capability and inability to provide a general solution for light transport effects.

Adaptive sampling and denoising filtering methods are widely used in the industry. Pixar's RenderMan is an example in the movie industry that uses new statistical adaptive sampling metrics for generating high quality images with less effort and this idea has been used in the production of Toy Story 4 (RenderMan22.5, 2019; fxguide, 2020). Gaming or medical imaging industries are other fields looking for adaptive sampling and denoising filtering based solutions. Gaming industries are interested in techniques that enables generating high quality images with low samples (Kuznetsov et al., 2018), and medical imaging companies use these methods in the evaluation of Magnetic Resonance Images (MRIs) (Russo, 2010).

In this work, state-of-the-art adaptive sampling and denoising filtering methods are investigated. In addition, the challenges of having a general denoising filter for a wide range of effects or the problem of representing complex effects are addressed in this study. Finally, this paper offers some solutions for further investigation that can be followed representing complex effects.

2 Analysis of Adaptive Sampling and Denoising Filtering Methods

This section contains two subsections related to the work improving Monte Carlo rendering process. In the first subsection, the work on adaptive sampling methods is analyzed. In the second subsection, we analyze denoising filtering methods.

2.1 Adaptive Sampling

In traditional Monte Carlo path tracers, a fixed number of samples per pixel was traced in the applications. However, each part of the scene does not have the same complexity where some parts may need fewer samples and some may need more samples to converge in the computation. Therefore, adaptive sampling strategy emerged to prevent this waste. The idea behind this method is using minimum number of samples per pixel, and increase the number of samples only for the parts that have a contrast value above the initially defined threshold value. Although the metrics and evaluation techniques may vary for different samplers, the general workflow is visualized as a diagram in Figure 1 given by RenderMan22.5. (2019).

Adaptive sampling methods have been studied by researchers for a long period of time. An earlier work which inspired researchers in this field is the work of Mitchell (1987)(Zwicker et al., 2015). In this study, Mitchell describes an antialiasing module to define the sampling procedure considering the noise perception. Ward et al. (1998) proposed a greedy irradiance caching algorithm for interpolating irradiance values in ray tracing applications. Painter & Sloan (1989) and Guo (1998) followed progressive refinement for sampling the image plane. Other earlier research

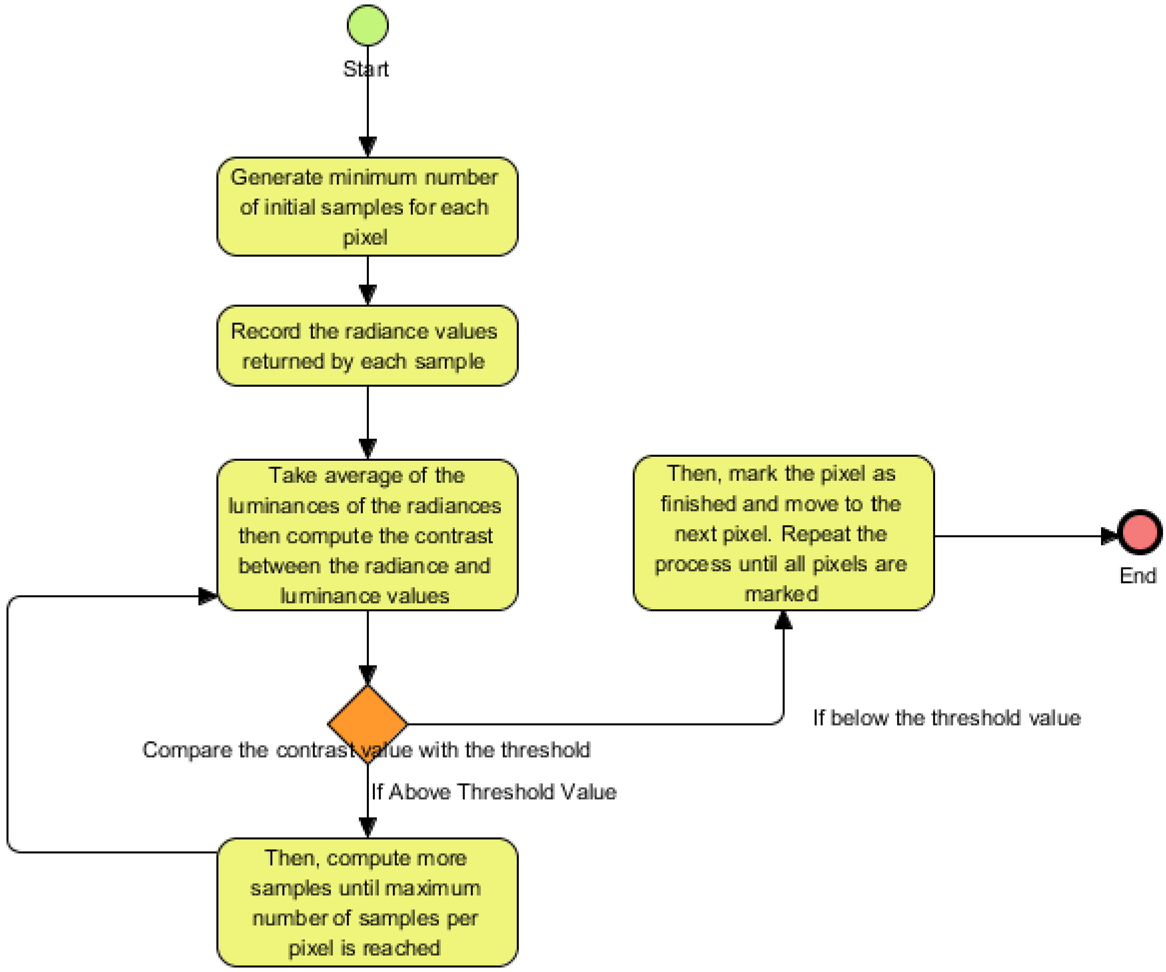

Figure 1: The general flowchart of the adaptive sampling methods (RenderMan22.5., 2019).

can be given as Bolin & Meyer (1998) which defines a metric for the sample distribution and represents a Haar wavelet transform of the data to be used for image synthesis.

Recently, more advanced approaches have been proposed in adaptive sampling that can preserve the quality of the images without reducing the number of samples to be used in the path tracing computation. These works are based on analyzing the local light transport equations or controlling the sample information provided by the Monte Carlo rendering operation. As a pioneer work, Durand et al. (2005) made an analysis of local frequencies of light transport considering individual rays. Similar work was followed by Levin et al. (2009); Wetzstein et al. (2011) for computational cameras and light field displays. In the work of Hachisuka et al. (2008), a multidimensional adaptive sampling strategy was proposed in which the samples are distributed in the full sampling domain. This work inspired many other research studies that work in higher dimensional space. However, it ignores the underlying effects resulting in less quality in the final image. Moreover, the consideration of more effects leads to higher dimensions which become computationally inefficient. As a significant study, Overbeck et al. (2009) adaptively distributed samples in the image space considering the variance of the samples performing a wavelet analysis of the final image. After this process, the image is reconstructed using an appropriate wavelet approximation.

The effects of depth of field, motion blur, soft shadows and directional occlusions were also performed using adaptive sampling methodology (Soler et al., 2009; Egan et al., 2009; Egan et al., 2011a; 2011b. Ringing artifacts caused by sampled wavelets was addressed by Rousselle et al. (2011) that introduces a distribution of new samples for minimizing the mean squared error. An example "sibenik" scene was denoised using their algorithm and shown in Figure 2. Lehtinen et al. (2011) exploited light field shearing handling soft shadows and motion blurs. Some interactive methods have been proposed considering the reconstruction scheme that can be given as Mehta et al. (2014); Yan et al. (2016) presenting axis-aligned and 4D sheared filtering, which are computationally efficient. However, these approaches are lacking in representing some of the rendering details. Herholz et al. (2016) integrated importance sampling with the bidirectional scattering distribution function (BSDF) that gives better convergence in scenes with complex illumination. Vorba & Křivánek (2016) estimated the expected contribution of a light path using Russian Roulette and splitting techniques. Müller et al. (2017) used a combined data structure of spatial and directional domain to represent importance sampling for simulating caustics. A final work in this section is the work of Huo et al. (2020) which used an offline dataset of adaptive samples and reconstructs the incident radiance field using deep reinforcement learning.

For further analysis on adaptive sampling, Zwicker et al. (2015) presented a comprehensive study covering adaptive sampling based studies and applications which can be viewed for more information.

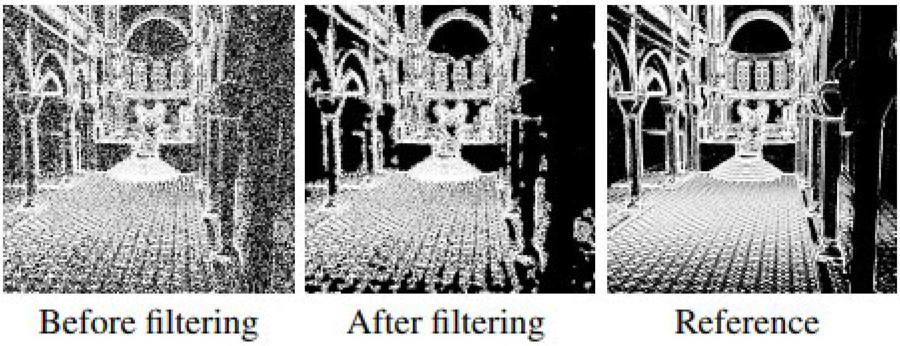

Figure 2: Rousselle et al. (2011) visualized post-processing procedure using a 2D "sibenik" scene example showing the stopping map from scale 2 to 3. The input data consists of 32 noisy samples per pixel (left) and the post-processed image does not lose features (middle) compared to the reference image (right) shared in the figure.

2.2 Denoising Filtering

Denoising filters are the approaches that are performed as a post-processing strategy. The idea is to use the information gathered from the rendering process and employ a direct reconstruction scheme for removing the noisy pixels in the final image. The flowchart of this process is shown in Figure 3. Denoising filters are very popular due to their mathematical simplicity and efficiency. The majority of these filters can be stated as general bilateral filters (Paris et al., 2007; Delbracio et al., 2014). However, the complexity of each denoising filter depends on the information that it is processing, which may differ from one to another. It is also a fact that some filters operate at pixel level and some of them use the additional sampling information provided by the renderer. This also changes the complexity of the proposed approach which is also discussed in this subsection.

As denoising filters are applied on image space, there is a number of quality metrics that can be used in the evaluation of the denoised images. Popularly used metrics can be named as Mean Squared Error (MSE), peak-signal-to-noise ratio (PSNR) or signal-to-noise ratio (SNR) which are also commonly used by researchers (Zhang et al., 2012). MSE computes the L_2 norm of the arithmetic difference between the reference and test image. It is stated that PSNR is more useful than MSE when we have a set of dynamic ranged images. However, PSNR is still closely related with MSE and both of the metrics cannot perform to evaluate content-dependent variations. On the other hand, SNR is taking mean of the intensities into consideration. However, SNR also fails to evaluate the image quality accurately in many cases. In addition, none of these metrics are capable of evaluating filters with contrast enhancement. Therefore, researchers have proposed other metrics for better evaluation of the filters. One of these metrics is Homogeneity Mean Difference (HMD) which includes the components of edge value, standard deviation and entropy in the characterization of the evaluation process. These components define the homogeneity of the test image and the metric can accurately evaluate the quality image as well as complex effects which were ignored by the popular metrics (Zhang et al., 2012). Nevertheless, different metrics can perform better in different conditions. As a result, it is important to choose the most appropriate metric for the specified process.

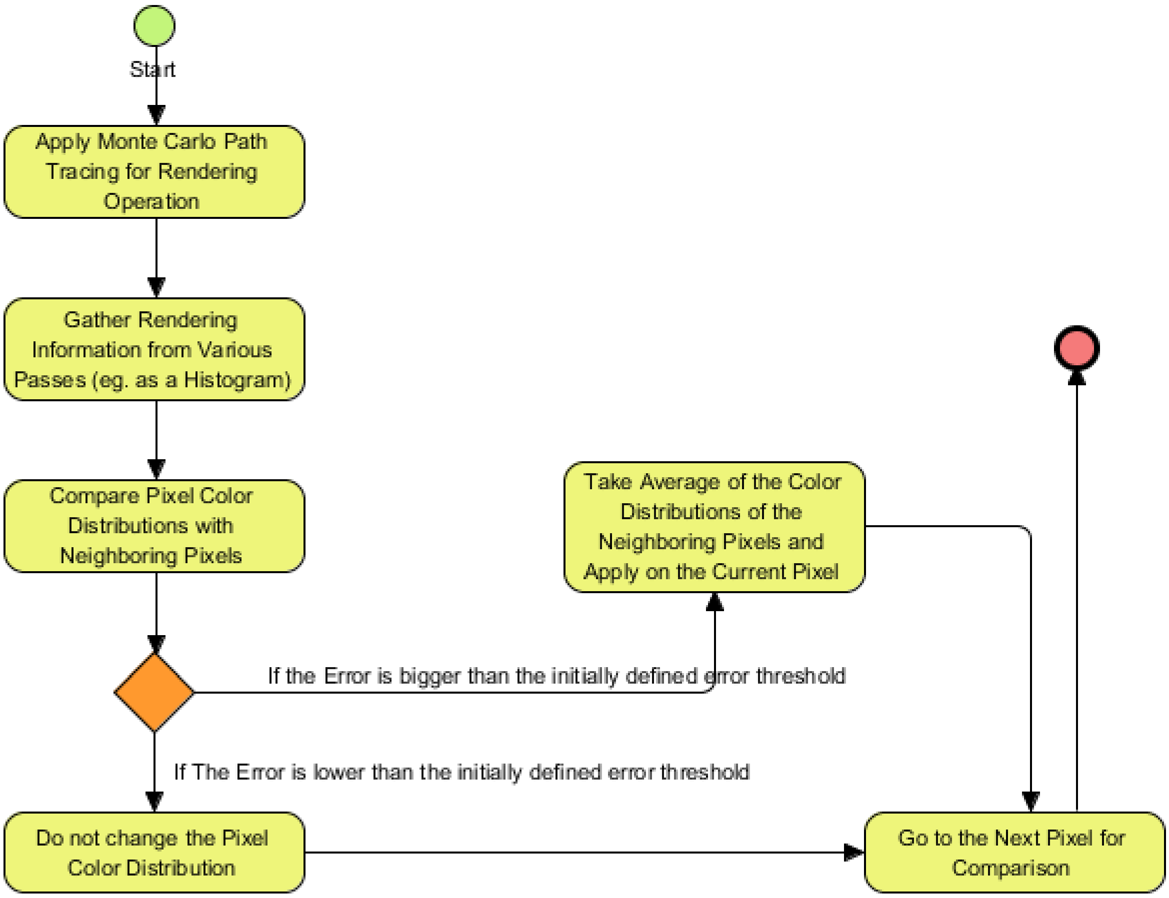

Figure 3: The general flowchart of the denoising filtering methods (Delbracio et al., 2014)

An earlier work classified in this group is presented by Lee & Redner (1990) that defines an alpha trimmed filter. Jensen & Christensen (1995) proposed using Gaussian or median filters for the light diffusely reflecting multiple times. Choudhury & Tumblin (2003) introduced a trilateral filter and Xu & Pattanaik (2005) presented a classical bilateral image filter. The common property of these approaches is acting at pixel level and they are similar to the filters that have been widely used in the image processing field.

There are also denoising filters that operate at pixel level but include analysis of the sample information. Earlier examples are the work of Rushmeier & Ward (1994) which introduced image space nonlinear filters and McCool (1999) that introduced using anisotropic diffusion in the denoising operation. Dammertz et al. (2010) presented a real time filter which used wavelet transforms to acquire denoising. A bilateral filter containing weights of gradients were proposed by Xu et al. (2011).

A more complex group of filtering algorithms relies on using additional statistical information to filter the sample values. The members of this group can produce more qualified images than the previous groups. However, the complexity also increases due to the processed information. Shirley et al. (2011) presented a denoising filter considering the noise in motion blurred regions. A pioneer work in this group is proposed by Sen & Darabi (2012). Sample information and the scene features are processed in their work and their algorithm can denoise images even at low sample rates. However, the complexity of the algorithm becomes huge especially when larger number of samples are used in the filtering. There are also other approaches that handle the information gathered from auxiliary buffers (Li et al., 2012; Rousselle et al., 2013; Moon et al., 2017).

Another approach is using leveraging statistics which may include histograms and covariance matrices. Rousselle et al. (2012) presented a non-local means filter that determines the weight based on the patch similarity of the neighboring pixels. Delbracio et al. (2014) followed a similar approach in which sample histograms are stored during the rendering process and the patch similarity is computed by chi-square test which defines the difference of these histograms. According to their statements, this approach improves the results compared to non-local means filtering. Boughida & Boubekeur (2017) combined the histogram based filtering with non-local bayes filtering to improve denoising quality. As a pioneer work, Bitterli et al. (2016) proposed a novel approach that combines the individual elements from the subset of algorithms that they have chosen. Apart from these work, a number of notable studies have been proposed including estimation of gradients for temporal filtering (Schied et al., 2018), adaptive polynomial rendering (Moon et al., 2016) and a real time spatio-temporal filtering framework that is built on a hybrid ray tracer (Mara et al., 2017). Mara et al. (2017) also provided an interesting comparison scene that shows the results of the approaches that leverage statistical information which can be seen from Figure 4. Mara et al. (2017) presented that, the livingroom scene was denoised with 4 different approaches. The upper left image is the result of their proposed approach, the upper image was the output of Bitterli et al. (2016), and the bottom images were denoised by the approaches of Rousselle et al. (2012) (left) and Moon et al. (2014) (right). According to their results, the scene was rendered with a resolution of 1280x720x16 and the fastest approach was their work (ending in 0.31 seconds). Other images were denoised in 58 (Rousselle et al., 2012), 50 (Moon et al., 2014) and 119 (Bitterli et al., 2016) seconds, respectively. For further information about the quality metrics, the study of Mara et al. (2017) can be viewed.

A number of studies have been proposed using learning based strategies. Neural networks and deep learning algorithms were employed for enabling the learning process (Kalantari et al., 2015; Bako et al., 2017; Chaitanya et al., 2017; Xu et al., 2019; Gharbi et al., 2019; Huo et al., 2020). Another interesting approach is based on deep images which include the depth data compared to standard flat images. Vicini et al. (2019) presented a combined model of a flat image space non-local means filter with a deep cross bilateral filter which is an example study of this group.

For more information, the study of Goyal et al. (2020) can be viewed which covers the latest approaches proposed in the field of denoising filtering covering applications in various domains.

3 Challenges in Monte Carlo Denoising

As it can be seen from previous sections, Monte Carlo Denoising approaches have attracted the interest of researchers for a long period of time. Many models have been proposed in this manner, that try to find a general solution for complex effects in path tracing applications. Nevertheless, denoising is still an open research area since the proposed models in the literature is either computationally inefficient or they are not compatible with some of the complex effects. Moreover, the attraction of the researchers shifted towards robustness in order to produce accurate results without losing any rendering information and also achieving an acceptable efficiency at the same time (Vicini et al., 2019).

(a) Noisy "livingroom" input scene **(b)** Denoised image by 4 state-of-the-art methods

Figure 4: Denoising the sample "livingroom" image using 4 different methods proposed in the literature (Mara et al., 2017; Bitterli et al., 2016; Rousselle et al., 2012; Moon et al., 2014). According to the results, the work of Mara et al. (2017) is superior compared to others.

The Monte Carlo path tracing algorithm is based on the solution of the rendering integral proposed by Kajiya (1986):

$$L\left(x, \omega_o\right) = L_E\left(x, \omega_o\right) + L_R\left(x, \omega_o\right) \tag{1}$$

$$L_R\left(x, \omega_o\right) = \int_{\Omega} \rho\left(x, \omega_o, \omega_i\right) L\left(x, \omega_i\right) \cos\theta d\omega_i \tag{2}$$

In eq (1), L represents the outgoing radiance which is the sum of the emitted radiance (L_E) and the reflected radiance (L_R). Therefore, the rendering equation defines how the light is transported in the scene by defining the amount of energy being reflected and emitted based on the material properties. In the computation of L_R, we use bidirectional scattering distribution functions (BRDFs) which is represented as ρ on the hemispherical domain Ω in eq (2) (Herholz et al., 2016). In path tracing applications, each light path is built by sampling ω_i considering a probability density function (pdf) p where $p : \Omega \to \mathbb{R}$. As a result, L_R can be estimated as:

$$L'_R = \frac{\rho\left(x, \omega_o, \omega_i\right) L\left(x, \omega_i\right) \cos\theta}{p\left(\omega_i\right)} \tag{3}$$

The need for denoising approaches is related with the convergence rate of the Monte Carlo integration. The integration is solved in the rendering process and the integration can only converge when an appropriate number of samples are provided for the operation. Otherwise, the output image contains noisy pixels as it was previously explained. Considering eq (3), we can say that p should match the integrand in the ideal case, which would be dependent on the estimator having minimal variance value (Herholz et al., 2016). Therefore, if the p value is not close to the optimal value, there is a need for more samples in the computation or a post-processing operation for taking the average of neighboring pixels to replace it with the noises in the image space.

The main objective of the denoising approaches is to solve this problem without sacrificing from the performance. According to the effects focused in the rendering, different approaches can be suitable for this process. However, the complexity increases rapidly when complex effects are simulated. In addition, adaptive sampling and post-processing methods have different drawbacks which may result in blurs and artifacts. Therefore, the trend in this field is to present novel approaches including learning based algorithms and deep compositing.

Another problem is the parametrization of the proposed approach. For state-of-the-art work, these parameters are manually set for each task. It is also a fact that optimizations may be needed to be performed in the test cases. Consequently, the flexibility of these models is in debate.

It is also important that, especially for the post-processing based approaches, the rendering information gathered from the rendering process is crucial in the denoising operation. The capability of the algorithm depends on this data, which is used to define similarities and differences between the patches that will be used to denoise the noisy pixels. Therefore, the details of the rendering framework also becomes important since any missing data may lead to loss after post-processing. An example can be the case where depth of field effect is focused. In this case, the depth of field introduces noise in the GBuffer, and computing a GBuffer may lead to unrealistic results at the end (Boughida & Boubekeur, 2017).

The solution of these problems may be solved using learning based approaches. With the increasing capability of machine learning and deep learning algorithms, it is possible to learn the image and direction spaces as well as guiding the operation for better sampling. It is also important to provide an appropriate amount of datasets which will be used in the learning process of the proposed approach. Therefore, considering the unsupervised environment, these datasets can be used to train the network that will be effective to solve complex problems (Huo et al., 2020). However, currently there is a need for having a general, efficient and powerful learning algorithm. This is related with the fact that currently proposed algorithms are poorly generalized and especially for the deep learning methods, the memory requirements are increasing considering their deep structure (Vicini et al., 2019). Moreover, the training datasets are not available at the moment which is also required for better results in the future. As these datasets are crucial in the learning scheme, biased rendering techniques are not compatible with the nature of deep learning algorithms since they may contain inconsistencies. Therefore, dataset generation is strictly limited with unbiased rendering techniques. Nevertheless, the capability of these algorithms are worthwhile to investigate which is believed that such problems may be solved as more attention is shifted towards them. Moreover, path tracing is originally unbiased which is compatible with the deep learning dataset generation.

4 Conclusion

In this paper state-of-the-art adaptive sampling and denoising filtering approaches and a summary of the challenges in Monte Carlo denoising has been presented and discussed in detail. As it can be seen, a general solution is not available in denoising and many models have been proposed focusing on different effects. This paper also expresses some challenges are observed in this field and possible solution strategies that may be followed to overcome those problems.

Although the subject has been extensively studied for over than three decades, there is still a need of pioneering work for having more efficient renderers in the future. Especially with the improvements in deep learning, it is believed that it will dominate the future of denoising operations. Although there are some datasets available for open access (Brummer & De Vleeschouwer, 2019), in many fields there is still a need for more datasets for further progress (Li et al., 2016; Yong et al., 2019). Therefore, a general training database is needed to enhance developments in this field which can contain many datasets from various fields to be used in a wide range of applications. Such a database can be used for accurately training deep learning networks and also may be used as the beginning point to find a general solution.

5 Acknowledgement

The author would like to thank Ahmet Bilgili for his valuable help and supervision on denoising approaches which has an important role in the preparation of this review.

References

Bako, S., Vogels, T., Mcwilliams, B., Meyer, M., Novák, J., et al. (2017, July). Kernel-predicting convolutional networks for denoising monte carlo renderings. *ACM Trans. Graph., 36*(4).

Bitterli, B., Rousselle, F., Moon, B., Iglesias-Guitián, J. A., Adler, et al. (2016). Nonlinearly weighted first-order regression for denoising monte carlo renderings. In *Computer graphics forum* (Vol. 35, pp. 107-117).

Bolin, M.R., Meyer, G.W. (1998). A perceptually based adaptive sampling algorithm. In *Proceedings of the 25th Annual Conference on Computer Graphics and Interactive Techniques* (pp. 299-309). New York, NY, USA: Association for Computing Machinery.

Boughida, M., Boubekeur, T. (2017, July). Bayesian collaborative denoising for monte carlo rendering. In *Comput. Graph. Forum*, *36*(4), 137-153.

Brummer, B., De Vleeschouwer, C. (2019, June). Natural image noise dataset. In *The IEEE Conference on Computer Vision and Pattern Recognition* (CVPR) Workshops.

Chaitanya, C.R.A., Kaplanyan, A.S., Schied, C., Salvi, M., Lefohn, A., Nowrouzezahrai, D., & Aila, T. (2017, July). Interactive reconstruction of monte carlo image sequences using a recurrent denoising autoencoder. *ACM Trans. Graph., 36*(4).

Choudhury, P., Tumblin, J. (2003). The trilateral filter for high contrast images and meshes. In *Proceedings of the 14th eurographics workshop on rendering* (p. 186196). Goslar, DEU: Eurographics Association.

Dammertz, H., Sewtz, D., Hanika, J., & Lensch, H.P.A. (2010). Edge-avoiding À-trous wavelet transform for fast global illumination filtering. In *Proceedings of the conference on high performance graphics* (p. 6775). Goslar, DEU: Eurographics Association.

Delbracio, M., Musé, P., Buades, A., Chauvier, J., Phelps, N., & Morel, J.-M. (2014, February). Boosting monte carlo rendering by ray histogram fusion. *ACM Trans. Graph., 33*(1), 8:1-8:15.

Durand, F., Holzschuch, N., Soler, C., Chan, E., & Sillion, F.X. (2005, July). A frequency analysis of light transport. *ACM Trans. Graph.*, *24*(3), 1115-1126.

Egan, K., Durand, F., & Ramamoorthi, R. (2011a, December). Practical filtering for efficient ray-traced directional occlusion. *ACM Trans. Graph., 30*(6), 1-10.

Egan, K., Hecht, F., Durand, F., & Ramamoorthi, R. (2011b, April). Frequency analysis and sheared filtering for shadow light fields of complex occluders. *ACM Trans. Graph., 30*(2).

Egan, K., Tseng, Y.-T., Holzschuch, N., Durand, F., & Ramamoorthi, R. (2009, July). Frequency analysis and sheared reconstruction for rendering motion blur. *ACM Trans. Graph., 28*(3).

fxguide. (2020, April). *Renderman - what's new?* Retrieved from https://renderman.pixar.com/whats-new

Gharbi, M., Li, T.-M., Aittala, M., Lehtinen, J., & Durand, F. (2019, July). Sample-based monte carlo denoising using a kernel-splatting network. *ACM Trans. Graph., 38*(4).

Goyal, B., Dogra, A., Agrawal, S., Sohi, B., & Sharma, A. (2020). Image denoising review: From classical to state-of-the-art approaches. *Information Fusion*, 55, 220-244.

Guo, B. (1998). Progressive radiance evaluation using directional coherence maps. In *Proceedings of the 25th Annual Conference on Computer Graphics and Interactive Techniques* (p. 255-266). New York, NY, USA: Association for Computing Machinery.

Hachisuka, T., Jarosz, W., Weistroffer, R.P., Dale, K., Humphreys, G., Zwicker, M., & Jensen, H.W. (2008, August). Multidimensional adaptive sampling and reconstruction for ray tracing. *ACM Trans. Graph., 27*(3), 1-10.

Herholz, S., Elek, O., Vorba, J., Lensch, H., & Křivánek, J. (2016, July). Product importance sampling for light transport path guiding. *Comput. Graph. Forum, 35*(4), 67-77.

Huo, Y., Wang, R., Zheng, R., Xu, H., Bao, H., & Yoon, S.-E. (2020, January). Adaptive incident radiance field sampling and reconstruction using deep reinforcement learning. *ACM Trans. Graph., 39*(1).

Jensen, H., Christensen, N. (1995). Optimizing path tracing using noise reduction filters. In *Proceedings of WSCG95* (pp. 134-142).

Kajiya, J.T. (1986, August). The rendering equation. *SIGGRAPH Comput. Graph.*, *20*(4), 143-150.

Kalantari, N.K., Bako, S., & Sen, P. (2015, July). A machine learning approach for filtering Monte Carlo noise. *ACM Trans. Graph., 34*(4).

Kuznetsov, A., Kalantari, N.K., & Ramamoorthi, R. (2018). Deep adaptive sampling for low sample count rendering. In *Computer Graphics Forum* (Vol.37, pp.35-44).

Lee, M.E., Redner, R.A. (1990, May). Filtering: A note on the use of nonlinear filtering in computer graphics. *IEEE Comput. Graph. Appl., 10*(3), 23-29.

Lehtinen, J., Aila, T., Chen, J., Laine, S., & Durand, F. (2011, July). Temporal light field reconstruction for rendering distribution effects. *ACM Trans. Graph., 30*(4).

Levin, A., Hasinoff, S.W., Green, P., Durand, F., & Freeman, W.T. (2009, July). 4D frequency analysis of computational cameras for depth of field extension. *ACM Trans. Graph., 28*(3).

Li, T.-M., Wu, Y.-T., & Chuang, Y.-Y. (2012, November). Sure-based optimization for adaptive sampling and reconstruction. *ACM Trans. Graph., 31*(6).

Li, Y., Zheng, W., Cui, Z., & Zhou, X. (2016). A novel graph regularized sparse linear discriminant analysis model for eeg emotion recognition. In *International conference on neural information processing* (pp. 175-182).

Mara, M., McGuire, M., Bitterli, B., & Jarosz, W. (2017). An efficient denoising algorithm for global illumination. In *Proceedings of High Performance Graphics*. New York, NY, USA: Association for Computing Machinery.

McCool, M.D. (1999, April). Anisotropic diffusion for monte carlo noise reduction. *ACM Trans. Graph., 18*(2), 171-194.

Mehta, S.U., Yao, J., Ramamoorthi, R., & Durand, F. (2014, July). Factored axis-aligned filtering for rendering multiple distribution effects. *ACM Trans. Graph., 33*(4).

Mitchell, D.P. (1987). Generating antialiased images at low sampling densities. In *Proceedings of the 14th Annual Conference On Computer Graphics And Interactive Techniques* (pp. 65-72). New York, NY, USA: Association for Computing Machinery.

Moon, B., Carr, N., & Yoon, S.-E. (2014, September). Adaptive rendering based on weighted local regression. *ACM Trans. Graph., 33*(5).

Moon, B., Iglesias-Guitian, J. A., McDonagh, S., & Mitchell, K. (2017). Noise reduction on g-buffers for monte carlo filtering. In *Computer Graphics Forum* (Vol. 36, pp.600-612).

Moon, B., McDonagh, S., Mitchell, K., & Gross, M. (2016, July). Adaptive polynomial rendering. *ACM Trans. Graph., 35*(4).

Müller, T., Gross, M., & Novák, J. (2017, July). Practical path guiding for efficient lighttransport simulation. *Comput. Graph. Forum, 36*(4), 91-100.

Overbeck, R.S., Donner, C., & Ramamoorthi, R. (2009, December). Adaptive wavelet rendering. *ACM Trans. Graph., 28*(5), 1-12.

Painter, J., Sloan, K. (1989, July). Antialiased ray tracing by adaptive progressive refinement. *SIGGRAPH Comput. Graph., 23*(3), 281-288.

Paris, S., Kornprobst, P., Tumblin, J., & Durand, F. (2007). A gentle introduction to bilateral filtering and its applications. In *ACM SIGGRAPH 2007 courses* (p. 1es). New York, NY, USA: Association for Computing Machinery.

Pharr, M., Humphreys, G. (2010). *Physically based rendering, second edition: From theory to implementation* (2nd ed.). San Francisco, CA, USA: Morgan Kaufmann Publishers Inc.

RenderMan22.5. (2019, May). R*enderman 22.5 ships: Toys go wild!* Retrieved from https://fxguide.com/fxfeatured/renderman-22-5-ships-toys-go-wild/

Rousselle, F., Knaus, C., & Zwicker, M. (2011, December). Adaptive sampling and reconstruction using greedy error minimization. *ACM Trans. Graph., 30*(6), 1-12.

Rousselle, F., Knaus, C., & Zwicker, M. (2012, November). Adaptive rendering with non-local means filtering. *ACM Trans. Graph., 31*(6). Retrieved from https://doi.org/10.1145/2366145.2366214 doi: 10.1145/2366145.2366214

Rousselle, F., Manzi, M., & Zwicker, M. (2013). Robust denoising using feature and color information. In *Computer Graphics Forum* (Vol. 32, pp. 121-130).

Rushmeier, H.E., Ward, G.J. (1994). Energy preserving non-linear filters. In *Proceedings of the 21st annual Conference on Computer Graphics and Interactive Techniques* (pp. 131-138). New York, NY, USA: Association for Computing Machinery.

Russo, F. (2010). Validation of denoising algorithms for medical imaging. In *Advances in Biomedical Sensing, Measurements, Instrumentation and Systems* (pp. 93-105). Springer.

Schied, C., Peters, C., & Dachsbacher, C. (2018, August). Gradient estimation for real-time adaptive temporal filtering. *Proc. ACM Comput. Graph. Interact. Tech., 1*(2).

Sen, P., Darabi, S. (2012, May). On filtering the noise from the random parameters in monte carlo rendering. *ACM Trans. Graph.*, *31*(3).

Shirley, P., Aila, T., Cohen, J., Enderton, E., Laine, S., Luebke, D., & McGuire, M. (2011). A local image reconstruction algorithm for stochastic rendering. In *Symposium on interactive 3D Graphics and Games* (p. 914 PAGE@5). New York, NY, USA: Association for Computing Machinery.

Soler, C., Subr, K., Durand, F., Holzschuch, N., & Sillion, F. (2009, May). Fourier depth of field. *ACM Trans. Graph., 28*(2).

Vicini, D., Adler, D., Novák, J., Rousselle, F., & Burley, B. (2019). Denoising deep monte carlo renderings. In *Computer Graphics Forum* (Vol. 38, pp. 316-327).

Vorba, J., Křivánek, J. (2016, July). Adjoint-driven russian roulette and splitting in light transport simulation. *ACM Trans. Graph., 35*(4).

Ward, G.J., Rubinstein, F.M., & Clear, R.D. (1988, June). A ray tracing solution for diffuse interre ection. *SIGGRAPH Comput. Graph., 22*(4), 85-92.

Wetzstein, G., Lanman, D., Heidrich, W., & Raskar, R. (2011, July). Layered 3d: Tomographic image synthesis for attenuation-based light field and high dynamic range displays. *ACM Trans. Graph., 30*(4).

Xu, B., Zhang, J., Wang, R., Xu, K., Yang, Y.-L., Li, C., & Tang, R. (2019, November). Adversarial monte carlo denoising with conditioned auxiliary feature modulation. *ACM Trans. Graph., 38*(6).

Xu, Q., Liu, Y., Zhang, R., Bao, S., Scopigno, R., & Sbert, M. (2011). Noise reduction for path traced imaging of participating media. In *Proceedings of the 19th European Signal Processing Conference, EUSIPCO 2011*, Barcelona, Spain, August 29 - Sept. 2, 2011 (pp. 1683-1687). IEEE.

Xu, R., Pattanaik, S.N. (2005, March). A novel monte carlo noise reduction operator. I*EEE Comput. Graph. Appl., 25*(2), 31-35.

Yan, L.-Q., Mehta, S. U., Ramamoorthi, R., & Durand, F. (2016, December). Fast 4d sheared filtering for interactive rendering of distribution effects. *ACM Trans. Graph., 35*(1).

Yong, S., Kim, C., & Kim, J. (2019). *Data augmentation and model optimization for piano transcription.* Retrieved from $https : //www.skt.ai/research/detail.doseq = 37$

Zhang, Y., Cheng, H.-D., Huang, J., & Tang, X. (2012). An effective and objective criterion for evaluating the performance of denoising filters. *Pattern Recognition, 45*(7), 2743-2757.

Zwicker, M., Jarosz, W., Lehtinen, J., Moon, B., Ramamoorthi, R., Rousselle, F., et al. (2015, May). Recent advances in adaptive sampling and reconstruction for monte carlo rendering. *Comput. Graph. Forum, 34*(2), 667-681.