Journal of Modern Technology and Engineering
Vol.7, No.2, 2022, pp.124-133

A NOVEL FUZZY INFERENCE MODEL WITH RULE-BASED DEFUZZIFICATION APPROACH

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Abstract. Fuzzy inference system (FIS) is one of the most used approaches in decision making and machine learning models. The main advantage of this system is that it can process data in the fuzzy form whose exact value is unknown. Various FIS models are available in the literature. The most common FIS models are Mamdani and Sugeno type models. In Sugeno type FISs, the output of each rule should be given as a specific mathematical function. Sometimes these functions can be difficult to identify. In the Mamdani model, it is sufficient to give an approximate fuzzy set instead of the specific function as a rule output. The overall output of the Mamdani type model is the aggregated fuzzy set, which is an aggregation of individual rule outputs. But in this composite set, the contours of each rule output are lost and the effect on the overall output is not very obvious. A novel type FIS with rule-based defuzzification (FIS-RBD) is proposed in this study. This model is a Mamdani type FIS, however, the defuzzification process is applied to each rule's output instead of the overall output. The overall output of the system is calculated as the weighted average of the defuzzified values of the rule outputs. This approach is a synthesis of a Mamdani type and Sugeno type models. In the study, the details of the proposed approach are examined and the working principle is explained with numerical examples.

Keywords: Fuzzy inference system, Defuzzification methods, Fuzzy number, FIS-RBD.
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Received: 3 May 2022; Revised: 28 June 2022; Accepted: 10 July 2022; Published: 6 September 2022.

1 Introduction

The most widely used model of fuzzy logic is the Fuzzy Inference System (FIS) (Pedrycz, 1993; Tanaka & Wang, 2001; Nasiboglu, 2020). Commonly used types of fuzzy inference model are Mamdani type and Sugeno type models (Mamdani, 1974; Mamdani & Assilian, 1975; Sugeno, 1985). Since the outputs of the rules are in the form of functions in the Sugeno model, there is no need for defuzzification. But in the Mamdani type model, the outputs of the rules and the overall aggregated output of the model are generally in the form of fuzzy sets. When necessary, defuzzification methods are applied to calculate the final crisp output of the model.

There are various defuzzification methods such as Center of Gravity (COG), Mean of Maxima (MOM), Bisector of Area (BOA), Weighted Average Based on Levels (WABL) etc. (Leekwijck & Kerre, 1999; Jiang & Li, 1996; Roychowdhury & Pedrycz, 2001; Nasibov, 2002; Mallick & Das, 2021). The next section provides detailed information about these methods. Among these methods, the WABL method is a more universal method and can act like other methods if its parameters are adjusted properly (Nasibov & Mert, 2007; Mert, 2020; Nasiboglu & Abdullayeva, 2018). In the literature, there are studies that facilitate the adjustment of the parameters of the method and the calculation of WABL values in various situations (Nasibov & Mert, 2007; Nasiboglu & Abdullayeva, 2018, Nasiboglu & Erten, 2019).

In the classical Mamdani type fuzzy inference model, the overall output of the model is the fuzzy set consisting of the combination of the individual rule outputs. Generally, the contours of the fuzzy numbers that make up this set are lost in this composite set. Therefore, the originality of each rule output is lost in the last defuzzification process. Therefore, it is clear that defuzzification of the output of each rule separately and taking their weighted average as the overall output of the model can take into account each rule output more effectively. In this study, a novel Mamdani type FIS with rule-based defuzzification (FIS-RBD) is proposed. In the proposed model, the overall output of the model is calculated as the weighted average of the rules' defuzzified outputs.

In the rest of the article, detailed information about FIS and defuzzification methods are given in the second section. In section 3, the proposed new approach for FIS with rule-based defuzzification is given. In the next section 4, the proposed approach is explained with a numerical example. In the conclusion section, the important points of the study are emphasized and suggestions for future studies are given.

2 Preliminaries

2.1 Fuzzy inference model

The fuzzy inference model can be used for classification or regression purposes in case of fuzzy data. The general form of the fuzzy inference mechanism is as follows:

Rule: if $x = A$ *then* $y = B$

Fact: $x = \tilde{A} \approx A$

Result: $y = \tilde{B} \approx B$

Here, the "$\sim$" sign on the character indicates that the appropriate value is fuzzy, and the "$\approx$" sign means approximate equality. The approximate degree of equality of the two fuzzy A_1 and A_2 values can be calculated as follows:

$$\mu\left(A_1 \approx A_2\right) = \max_x \min\{A_1\left(x\right), A_2\left(x\right)\} \tag{1}$$

Here the functions $A_1\left(x\right)$ and $A_2\left(x\right)$ are membership functions of fuzzy numbers A_1 and A_2 accordingly.

The truncated fuzzy output of the fuzzy inference model for given fuzzy input $\tilde{A}\left(x\right)$ is calculated as follows:

$$\tilde{B}\left(y\right) = \min\{w, B\left(y\right)\}. \tag{2}$$

Here $B\left(y\right)$ is the whole consequent part of the rule and

$$w = \max_x \min\{\tilde{A}\left(x\right), A\left(x\right)\} \tag{3}$$

represents the firing degree of the fuzzy rule.

The Mamdani type fuzzy inference system is based on the fuzzy inference model given above. The working principle of the Mamdani FIS is given in the following pseudocode:

Step1. The firing degree of each rule is calculated.

Step 2. The consequent part of each rule is truncated at the level of firing degree of the appropriate rule.

Step 3. The truncated results of the rules are aggregated using the "max" operator to form the overall result of the FIS.

Step 4. Where necessary, the final output is calculated by defuzzifying the overall FIS result.

The general scheme of the FIS is given in Figure 1.

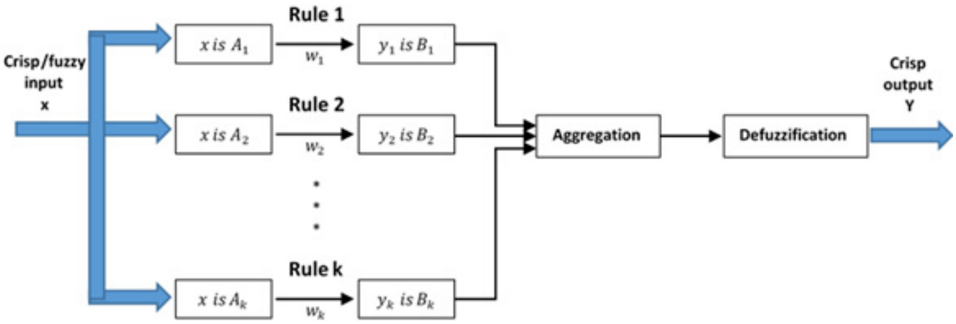

Figure 1: General scheme of the Fuzzy Inference System (FIS)

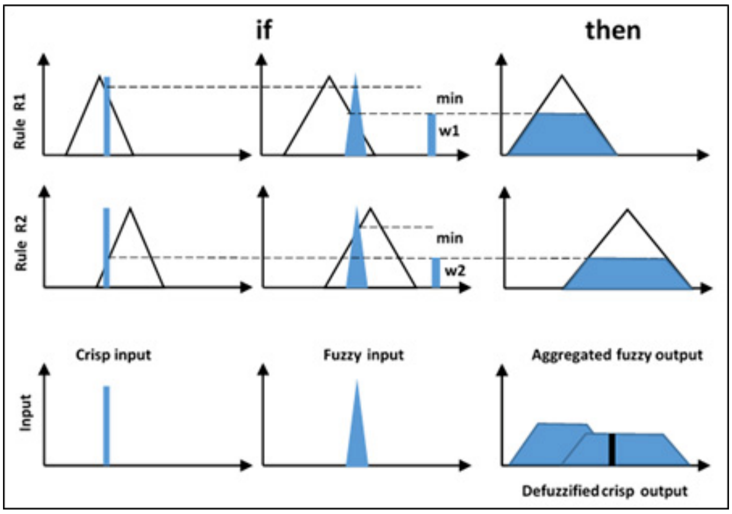

Figure 2: Example of a Mamdani type FIS with two rules and two inputs

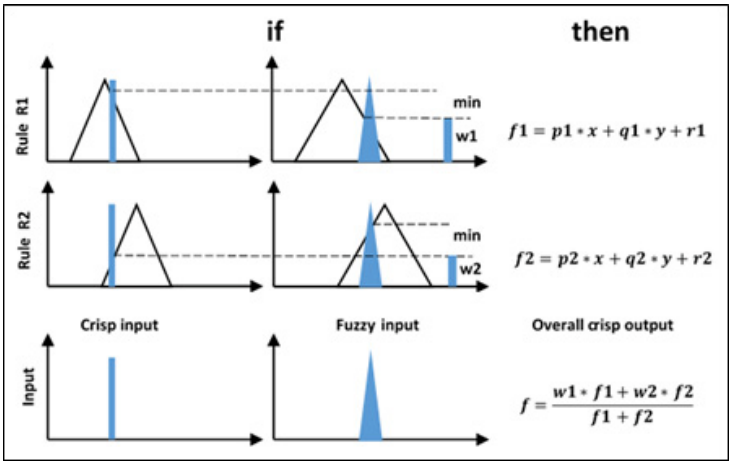

Figure 3: Example of a Sugeno type FIS with two rules and two inputs

There exists various types of fuzzy FIS models in the literature. The most widely used of these are the Mamdani and Sugeno type FISs. In the Mamdani FIS, the output of each rule is in the form of a fuzzy set. The overall output of the model is calculated as a combination of these fuzzy sets. As the last step, defuzzification is performed if it is required (Figure 2).

In the Sugeno type fuzzy inference model, the output of each rule is calculated as a function value. The overall output of the system is calculated as the weighted average of these values.

The weight of each rule's output is considered equal to the degree of firing of that rule (Figure 3).

2.2 Defuzzification methods

There are many application studies in the literature that give special attention to the defuzzification process (Sain & Mohan, 2021; Veerraju et al., 2020; Mendrek et al., 2018; Lukacs, 2019; Ezzati et al., 2014). The defuzzification process facilitates many fuzzy operations, but often also meets the system's exact output requirement (Roychowdhury & Pedrycz, 2001; Mallick & Das, 2021; Kantarci & Nasibov, 2017). MOM (Mean of Maxima) and COG (Centroid of Gravity) methods are among the most preferred defuzzification methods. These methods can be expressed in the form of mathematical formulas as follows:

$$MOM(A) = mean\{x : A\,(x) = 1\}, \tag{4}$$

$$COG(A) = \frac{\int_{-\infty}^{\infty} A\,(x)\,xdx}{\int_{-\infty}^{\infty} A\,(x)dx}. \tag{5}$$

where, $A(x)$ is the membership function of any given fuzzy number.

In formulas (4) and (5), the integral calculation for calculating the defuzzified value is done on the x-axis. However, in many cases integral calculations based on the level axis are performed (Pourabdollah et al., 2020; Mert, 2020; Nasibov, 2002, 2003, 2007; Nasibov & Shikhlinskaya, 2003). Among these methods, the WABL defuzzification approach is a more universal method and is used in several studies. Let $A = (L_A, R_A)$ be a fuzzy number given in LR-representation, where $L_A : [0,1] \to R^1$ and $R_A : [0,1] \to R^1$ are the left side and the right side functions, respectively. The WABL (Weighted Averaging Based on Levels) value of the fuzzy number A is calculated as follows:

$$WABL\,(A) = \int_0^1 ((1-c)\,L_A\,(t) + cR_A\,(t))\,p\,(t)\,dt. \tag{6}$$

Here, the parameter $c \in [0,1]$ reflects the importance of the left or right side of the fuzzy number (the optimism index). The $p : [0,1] \to R^1$ function, satisfying the following conditions, reflects the degree of importance of the level sets:

$$\int_0^1 p\,(t)\,dt = 1, p\,(t) \geq 0. \tag{7}$$

In the studies (Nasibov, 2002; Nasibov & Mert, 2007), the $p\,(t)$ function is determined parametrically as follows:

$$p(t) = (k+1)t^k. \tag{8}$$

The k parameter ($k \geq 0$) in formula (8) indicates the increasing rate of importance of the fuzzy number's level sets. If the parameters are set appropriately the WABL operator can act like the MOM or COG methods. In the study (Nasibov & Mert, 2007) the following theorems have been proved:

Theorem 1. *If the parameters of the WABL operator are set as*

$$c = 0.5,\ and\ p_\varepsilon\,(t) = \begin{cases} \frac{1}{\varepsilon}, & t \geq 1-\varepsilon, \\ 0, & t < 1-\varepsilon, \end{cases} \tag{9}$$

where $\varepsilon > 0$ is a sufficiently small value, then for any fuzzy number A it is satisfied:

$$MOM\,(A) = \lim_{\varepsilon \to 0} WABL_\varepsilon(A) \ = \int_0^1 ((1-c)\,L_A\,(t) + cR_A\,(t))\,p_\varepsilon\,(t)\,dt. \tag{10}$$

Theorem 2. *If the parameters of the WABL operator are set as*

$$c = \frac{r}{l+r}, \text{ and } p(t) = 2t, \tag{11}$$

then for any triangular fuzzy number A it is satisfied:

$$WABL(A) = COG(A) \tag{12}$$

Nasibov and Mert (2007), proved that for any fuzzy number $A = (m, l, r)$, where m is the center, and l and r are the left and right spreads of the fuzzy number respectively, the WABL value in the more general case can be calculated as follows:

$$WABL(A) = c\left((m+r) - \frac{k+1}{k+2}r\right) + (1-c)\left((m-l) + \frac{k+1}{k+2}l\right). \tag{13}$$

Here, c is the optimism parameter and k is the parameter in the formula (8). Specifically, in the case of $k = 0$, i.e. all levels are equally important, the formula (13) turns into the following form for any triangular fuzzy number:

$$WABL(A) = c\left(m + \frac{r}{2}\right) + (1-c)\left(m - \frac{l}{2}\right). \tag{14}$$

It should be noted that it does not require integral calculation in formulas (13) and (14), and therefore provides great convenience in calculating WABL values.

3 The proposed FIS with rule-based defuzzification (FIS-RBD)

In the proposed approach, instead of defuzzification of the fuzzy outputs of all rules after they are aggregated, the output of each rule is defuzzified and the overall output of the system is calculated as the weighted average of the rules' outputs. The weight of each rule output is equal to the degree to which that rule is fired. This approach is a mix of Mamdani type and Sugeno type FISs. However, the defuzzification method will have a greater effect, since the defuzzification process is applied to each rule output rather than to a single general output. Therefore, the overall system output can be changed more flexibly using parameter adjustments of the defuzzification method. The general schematic representation of this model is given in Figure 4.

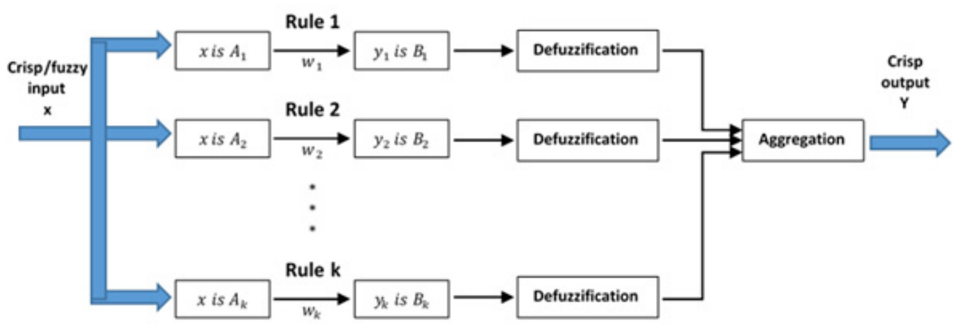

Figure 4: General scheme of the proposed RBD FIS

The rule base of the proposed model is assumed to be as follows:
Rule 1: If x_1 *is* A_{11} and x_2 *is* A_{12} and ... x_{n_1} *is* A_1n_1 then y_1 *is* B_1
Rule 2: If x_1 *is* A_{21} and x_2 *is* A_{22} and ... x_{n_2} *is* A_2n_2 then y_2 *is* B_2
...
Rule k: If x_1 *is* A_{k1} and x_2 *is* A_{k2} and ... x_{n_k} *is* A_kn_k then y_1 *is* B_k

Here A_{ij}, $i = 1,..,k; j = 1,\ldots,n_k$, are fuzzy numbers and represents the j.th input term of the i.th rule. The B_i value is the fuzzy output of the i.th rule.

In case of fuzzy $A(x)$ input, the degree of conformity of this input to fuzzy A_{ij} terms can be calculated as follows:

$$\mu\left(A\ is\ A_{ij}\right) = \max_x \min\{A\left(x\right), A_{ij}\left(x\right)\}\ , i = 1,..,k,\ j = 1,\ldots,n_i. \tag{15}$$

Here functions $A\left(x\right)$ and $A_{ij}\left(x\right)$ are membership functions of fuzzy numbers A and A_{ij} accordingly. The firing degree of the i.th rule is calculated as follows:

$$w_i = \min_{j=1,\ldots,n_i} \mu\left(A\ is\ A_{ij}\right)\ , i = 1,..,k. \tag{16}$$

The fuzzy output truncated by firing degree of the i.th rule is calculated as follows:

$$\widetilde{B_i}\left(y\right) = \min\{w_i, B_i\left(y\right)\}, i = 1,..,k. \tag{17}$$

Finally, the fuzzy output of the i.th rule is defuzzified:

$$y_i = Defuzzy(\widetilde{B_i}\left(y\right)), i = 1,..,k. \tag{18}$$

Here, WABL, COG, MOM etc. defuzzification operators can be used instead of the $Defuzzy(.)$ operation. The overall output of the system is calculated as follows:

$$y = \frac{\sum_{i=1}^{k} w_i y_i}{\sum_{i=1}^{k} w_i}. \tag{19}$$

The proposed FIS with rule-based defuzzification (FIS-RBD) is outlined in the following pseudocode:

Step 1. Repeat Step 2 to Step 4 for each rule:

Step 2. Calculate the degree of firing of each rule by formula (16).

Step 3. Calculate the truncated consequent part of each rule in accordance with formula (17).

Step 4. Defuzzify the fuzzy rule's result in accordance with the formula (18) and calculate the crisp value.

Step 5. Calculate the FIS overall result as the weighted average of the defuzzified rules' results in accordance with formula (19).

The detailed representation of this model is given in Figure 5.

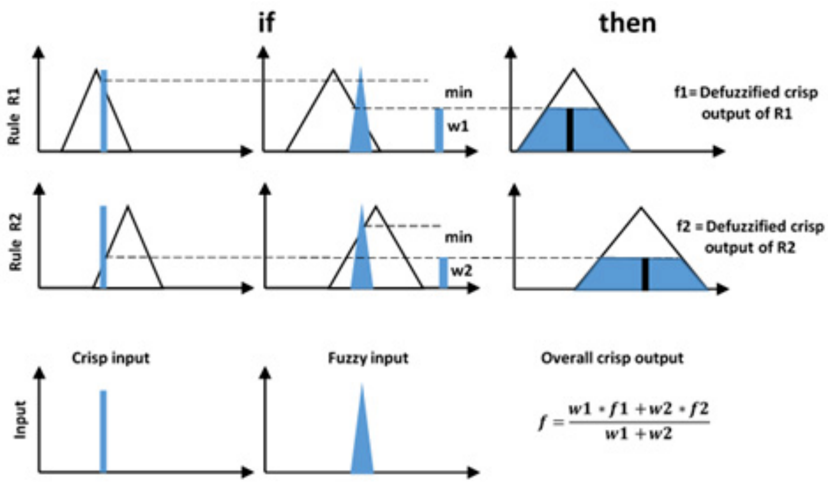

Figure 5: The proposed FIS-RBD with detailed rule-based defuzzification

4 Numerical examples

In this section, the working steps of the proposed fuzzy inference system will be explained through an example. For comparison, the same example will also be solved with the classic Mamdani FIS.

Let us given the following rule base:

Rule 1: ***if*** x *is* A_1 ***then*** y *is* B_1,

Rule 2: ***if*** x *is* A_2 ***then*** y *is* B_2,

with A_1= (1.0, 3.0, 5.0), A_2= (1.0, 5.0, 6.0), B_1= (1.0, 2.0, 5.0) and B_2= (3.0, 4.0, 6.0). Let's calculate the result of the system for the input x= 2.

Let's first do the calculations with the classic Mamdani type FIS. The resultant fuzzy output of the inference system will be calculated as follows (Figure 6).

For Rule 1: $w_1 = \mu_{A_1}(x) = 0.5$, accordingly, the truncated output of the Rule 1 is $\overline{B_1} = min\{w_1, B_1\}$.

For Rule 2: $w_2 = \mu_{A_2}(x) = 0.25$, accordingly, the truncated output of the Rule 1 is $\overline{B_2} = min\{w_2, B_2\}$.

Overall output of the FIS: $B = \overline{B_1} \cup \overline{B_2}$.

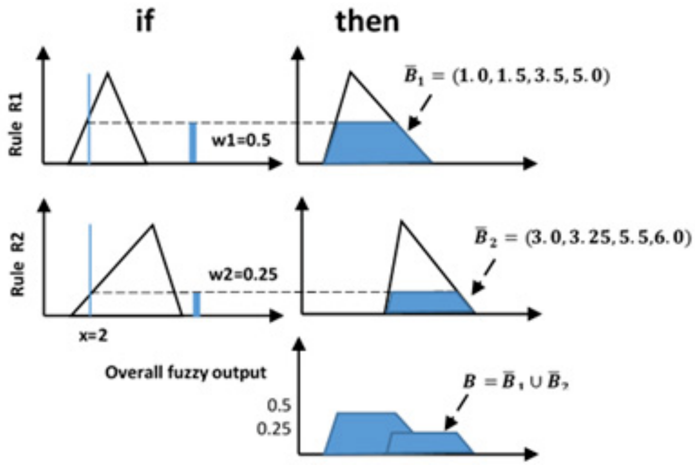

Figure 6: The classical Mamdani FIS according to the example

Let's calculate the LR functions of the terms used in the rules' outputs. It is clear that:

$$L_{\overline{B_1}}(t) = 1 + t, \quad 0 \le t \le 0.5,$$

$$R_{\overline{B_1}}(t) = 5 - 3t, \quad 0 \le t \le 0.5$$

$$L_{\overline{B_2}}(t) = 3 + t, \quad 0 \le t \le 0.25,$$

$$R_{\overline{B_2}}(t) = 6 - 2t, \quad 0 \le t \le 0.25.$$

So, overall output B of the FIS will be:

$$L_B(t) = 1 + t, \quad 0 \le t \le 0.5,$$

$$R_B(t) = \begin{cases} 6 - 2t, & 0 \le t \le 0.25, \\ 5 - 3t, & 0.25 \le t \le 0.5. \end{cases}$$

4.1 Calculations according to the WABL defuzzification

If we defuzzify the fuzzy FIS output with the WABL method with $c = 0.5$ and $p(t) = 1, \forall t \in [0,1]$, the overall defuzzified output of the system will be as follows:

$$WABL(B) = \frac{\int_0^{0.5} 0.5\left(L_B(t) + R_B(t)\right)p(t)dt}{\int_0^{0.5} p(t)dt} =$$

$$\int_0^{0.25}((1+t)+(6-2t))dt + \int_{0.25}^{0.5}((1+t)+(5-3t))dt =$$

$$\int_0^{0.25}(7-t)dt + \int_{0.25}^{0.5}(6-2t)dt = \left(7t - 0.5t^2\right)\Big|_0^{0.25} + \left(6t - t^2\right)\Big|_{0.25}^{0.5} = 3.3125 \tag{20}$$

Now let's calculate the results with the novel proposed FIS with the rule-based defuzzification approach.

$$WABL\left(\overline{B_1}\right) = \frac{\int_0^{0.5} 0.5\left(L_{\overline{B_1}}(t) + R_{\overline{B_1}}(t)\right)p(t)dt}{\int_0^{0.5} p(t)dt} =$$

$$\int_0^{0.5}((1+t)+(5-3t))dt = \left(6t - t^2\right)\Big|_0^{0.5} = 3 - 0.25 = 2.75$$

$$WABL\left(\overline{B_2}\right) = \frac{\int_0^{0.25} 0.5\left(L_{\overline{B_2}}(t) + R_{\overline{B_2}}(t)\right)p(t)dt}{\int_0^{0.25} p(t)dt} =$$

$$\frac{\int_0^{0.25} 0.5\left((3+t)+(6-2t)\right)dt}{0.25} = 2\left(9t - 0.5t^2\right)\Big|_0^{0.25} = 2(2.25 - 0.3125) = 3.875 \tag{21}$$

Finally, the overall crisp output of the system using WABL method will be as follows:

$$FIS\_RBD_{WABL}(x) = \frac{0.5 * 2.75 + 0.25 * 3.875}{0.5 + 0.25} = \frac{1.375 + 0.96875}{0.5 + 0.25} = 3.125 \tag{22}$$

4.2 Calculations according to the MOM defuzzification

Overall output of the example given above for the input $x = 2$ with classical Mamdani FIS using MOM defuzzification method will be as follows:

$$MOM(B) = \frac{1}{2}(1.5 + 3.5) = 2.5 \tag{23}$$

But if we use the FIS-RBD model, related calculations will be as follows:

For Rule 1: The defuzzified crisp output according to the MOM defuzzification method of the Rule 1 will be as follows:

$$MOM\left(\overline{B_1}\right) = \frac{1}{2}\left(L_{B_1}(0.5) + R_{B_1}(0.5)\right) = \frac{1}{2}(1.5 + 3.5) = 2.5 \tag{24}$$

For Rule 2: The defuzzified crisp output according to the MOM defuzzification method of the Rule 2 will be as follows:

$$MOM\left(\overline{B_2}\right) = \frac{1}{2}\left(L_{B_2}(0.25) + R_{B_2}(0.25)\right) = \frac{1}{2}(3.25 + 5.5) = 4.375 \tag{25}$$

The overall output of the FIS with rule based MOM defuzzification will be as follows:

$$FIS\_RBD_{MOM}(x) = \frac{w_1 MOM\left(\overline{B_1}\right) + w_2 MOM\left(\overline{B_2}\right)}{w_1 + w_2} = \frac{0.5 * 2.5 + 0.25 * 4.375}{0.5 + 0.25} = 3.125 \tag{26}$$

As can be seen from equations (20) and (23), the classical Mamdani FIS gives very different results when using the WABL and MOM defuzzification methods. However, as can be seen from the equations (22) and (26), the newly proposed FIS with rule-based defuzzification model gives closer and more consistent results.

5 Conclusion

In this study, a novel FIS model with rule-based defuzzification (FIS-RBD) is proposed as an alternative to the Mamdani type fuzzy inference system. Computational examples of the newly proposed model is given using different WABL and MOM defuzzification methods. In classical FIS, since the defuzzification is done on the last aggregated output, the left/right side information of the fuzzy number forming the output of each rule may be lost. However, since the output of each rule is defuzzified separately in the proposed FIS with rule-based defuzzification, this information can be used without loss. Therefore, the newly proposed model produces more consistent results and is more flexible than the classical Mamdani type FIS.

In future studies, we intend to investigate the FIS-RBD approach by considering the WABL defuzzification method more comprehensively and to address the problem of optimal tuning of parameters using a data-driven approach.

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