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ANALYSIS OF DIFFERENT APPROACHES TO REGRESSION PROBLEM WITH FUZZY INFORMATION

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Abstract. Approaches based on various technologies have been proposed in the literature for the creation of regression models, which are among the most widely used models in fields such as machine learning and data mining. In this article, various approaches such as mathematical programming, fuzzy distance-based least squares, fuzzy c-regression, fuzzy decision trees are discussed for constructing regression models with fuzzy information. Comparative analysis of these algorithms was made in terms of factors such as learning time, ease of processing of fuzzy data, and the effect of large data volume.

Keywords: Machine learning, Fuzzy regression, Fuzzy number, Fuzzy distance, Least squares.
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1 Introduction

Machine learning models are among the most used technologies recently. Among these models, estimation models are among the most preferred models. In estimation models, they are divided into regression and classification models depending on whether the predicted target variable is numerical or categorical. For example, estimating the amount of insurance premium is a regression problem. Whether a loan application is approved or not is a classification problem. The distinction here is made only according to the target variables. Input variables can be of any type in both problems.

Another point where estimation problems differ is that the variables are crisp and/or fuzzy. Especially in crisp (classical) classification problems, each output state can only belong to a single class, whereas in fuzzy classification, it can belong to different classes at the same time with different degrees. Algorithms that can work with a fuzzy approach are generally more robust and converge relatively faster (Nasibov & Ulutagay, 2009).

Regression analysis is a widely used methodology to analyze the relationships and correlations between a response variable, also called a dependent variable, and one or more explanatory variables, so called independent variables. For example, if past information is known about how many hours a student has studied for exams and how many points he has scored, it can be predicted a certain future exam score based on the student's study hours.

The classical linear regression model can be expressed as:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \cdots + \beta_n X_{in} + \varepsilon_i, \ for \ i = 1, 2, \ldots, m. \tag{1}$$

Here, Y_i is the value of the dependent (predicted) variable, β_j, $j = 0, \ldots, n$, are function coefficients, X_{ij}, are values of the independent variables, ε_i, are error terms. In the model, n denotes the number of independent variables and $i = 1, 2, \ldots, m$, denotes the observation index.

In the regression model, the cases where the dependent and/or independent variables can take fuzzy values and/or the cases where the regression function is a fuzzy function are called fuzzy regression model. Fuzzy regression models can be broadly divided into the following categories:

- Mathematical programming (MP) based approaches;
- Fuzzy least square errors (LSE) based approaches;
- Switching regression based approaches (FcRM);
- Fuzzy decision tree (DT) based approaches.

In this study, various approaches to fuzzy regression models will be discussed. In addition to explaining the models conceptually, their advantages and disadvantages will be mentioned.

2 Triangular fuzzy numbers and fuzzy operations

The fuzzy set is a subset defined by the membership function $\mu : X \to [0,1]$ in the universal set X. Fuzzy numbers are one of the most widely used fuzzy set forms in problems involving fuzzy information. Triangular fuzzy numbers (TFN) are often used in applications with fuzzy information. We will use the TFN, which membership function can be defined as follows (Nasiboglu & Nasibov, 2022, 2023):

Definition 1. *Triangular fuzzy number $A = (m, l, r)$ is a fuzzy number whose membership function is as follows:*

$$A(x) = \begin{cases} \frac{x-(m-l)}{l}, & m-l \le x \le m, \\ \frac{(m+r)-x}{r}, & m \le x \le m+r, \\ 0, & x \notin [m-l, m+r] \end{cases} \tag{2}$$

Here, m is the center (location index) of the fuzzy number, $l, r \ge 0$, are the left and right fuzziness spreads, respectively (Fig. 1).

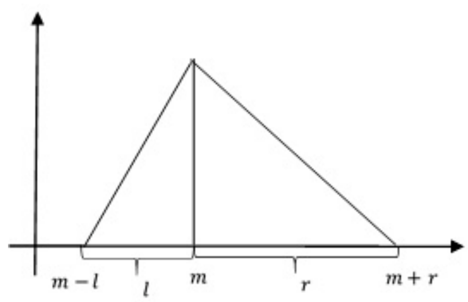

Figure 1: $A = (m, l, r)$ representation of a triangular fuzzy number.

Arithmetic operations on fuzzy numbers and the concept of defuzzification of fuzzy numbers have an important place in decision models based on fuzzy information. In the studies (Dubois & Prade, 1978; Goetschel & Voxman, 1986; Maa et al., 1999; Garg, 2018; Ngan, 2021; Seresht & Fayek, 2019), various definitions of arithmetic operations on fuzzy numbers have been proposed and applied.

Arithmetic operations on fuzzy numbers increase the width of the resulting fuzzy number too much. Especially when it comes to processing multiple fuzzy numbers in machine learning algorithms, the resulting fuzzy value spreads out meaninglessly. In order to prevent this situation, the approach suggested in the studies (Maa et al., 1999; Nasiboglu & Nasibov, 2022, 2023) can be used.

Definition 2. *The arithmetic operations on triangular fuzzy numbers* $A = (m_1, l_1, r_1)$ *and* $B = (m_2, l_2, r_2)$ *are defined as follows:*

$$A^{\circ}B = (m_1{}^{\circ}m_2, \max\{l_1, l_2\}, \max\{r_1, r_2\}), \tag{3}$$

Here, "$^{\circ}$" can be replaced by any operation "+", "-", "•" or "/".

3 Fuzzy Regression models

3.1 Mathematical Programming based approach

In mathematical programming (MP) based models, the deviation between the observed value and the predicted value of the dependent variable can be defined as "fuzzy" and depends on the fuzziness of the system structure. In other words, it is aimed to create a fuzzy function that its predicted values include the observed values of the dependent variable (Figure 2).

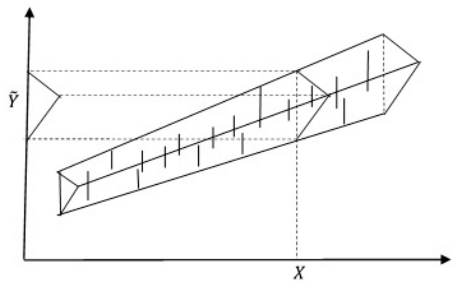

Figure 2: Tanaka's fuzzy regression model.

The linear programming (LP) based approach, which is a special case of MP based models, was first discussed in Tanaka (1982). In the study, the minimum value of the total fuzzy spreads of the coefficients was aimed as the objective function. The linear fuzzy regression model used in the study is as follows:

$$\tilde{Y}_i \in \tilde{Y}_i^* = \tilde{A}_0 + \tilde{A}_1 X_{i1} + \cdots + \tilde{A}_n X_{in},\ for\ i = 1, 2, \ldots, m. \tag{4}$$

Here $\tilde{Y}_i = (y_i, e_i)$, $i = 1, \ldots, m$, is the observed fuzzy value of the dependent variable. The X_i values taken by the independent variables are crisp values, but the coefficients $\tilde{A}_j = (a_j, c_j)$, $j = 0, 1, \ldots, n$ of the regression function consist of fuzzy numbers. In this case, the predictive values $\tilde{Y}_i^*$ of the function will also be fuzzy numbers. All fuzzy numbers considered in the model are symmetric triangular fuzzy numbers defined on the numerical axis $t \in R^1$. The membership functions of these fuzzy numbers are as follows:

$$\tilde{Y}_i(t) = (y_i, e_i) = \frac{|y_i - t|}{e_i}, \tag{5}$$

$$\tilde{A}_j(t) = (a_j, c_j) = \frac{|a_j - t|}{c_j}. \tag{6}$$

It is considered that the model proposed in (4) should satisfy the following basic conditions:

- It should minimize the total fuzzy spread of the parameters;
- The membership function of the predicted fuzzy value should include the membership function of the observed value (Figure 3).
- There should be a threshold parameter h, which shows the extent to which the predicted value fits the observed value (Figure 3).

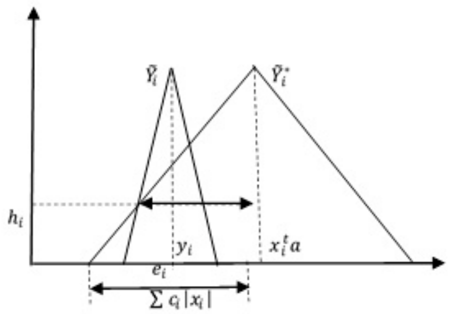

Figure 3: Estimated and observed fuzzy values.

- The fuzzy spread of a fuzzy number must not be negative.

In order to obtain the estimations of the parameters satisfying the above conditions, a linear programming model is proposed as follows (Tanaka, 1982):

$$\min z \ = c_0 + c_1 + \cdots + c_n, \tag{7}$$

Subject to:

$$\boldsymbol{a}^t \boldsymbol{x}_i + (1-h)\, \boldsymbol{c}^t\, |\boldsymbol{x}_i| \geq y_i + (1-h)e_i, \tag{8}$$

$$\boldsymbol{a}^t \boldsymbol{x}_i - (1-h)\, \boldsymbol{c}^t\, |\boldsymbol{x}_i| \leq y_i - (1-h)e_i, \tag{9}$$

$$c_i \geq 0, \ \ i = 0, 1, \ldots, n. \tag{10}$$

Here, the decision variables are the vectors $\boldsymbol{a}^t$ and $\boldsymbol{c}^t$, which specify the centers and spreads of the fuzzy coefficients of the regression function, respectively. Bolded $\boldsymbol{a}^t$, $\boldsymbol{c}^t$ and $\boldsymbol{x_i} = (1, x_0, x_1, \ldots, x_n)$ are vector values and the t sign above them indicates transpose has been received. (8) and (9) inequalities show that the observation value remains within the h level set of the fuzzy estimation value for $h \in [0, 1)$. (10) inequalities are the condition that the fuzzy spread values cannot be negative. The objective function in the formula (7) shows that the total fuzzy spread value should be as low as possible. Here, the h parameter can take values between 0 and 1. The closer the value of the h parameter is to 1, the closer the model is to the classical regression model, smaller values mean more fuzziness.

Later, the (7)-(10) model was developed and studies aiming at the minimum value of the total fuzziness of the products of the coefficients to the independent variables were carried out (Tanaka, 1987; Tanaka & Watada, 1988; Tanaka et al., 1989). In these studies, the constraint inequalities did not change, only the objective function was handled as follows:

$$\min z \ = \sum_{i=1}^{n} c_0 + c_1|x_{i1}| + \cdots + c_n|x_{in}|. \tag{11}$$

Various studies have also been carried out for some nonlinear cases. In the study (Tanaka & Ishibuchi, 1991), cases where the membership functions of fuzzy numbers are not linear, and in the study (Tanaka, 1998), the square fuzzy regression model, which minimizes the squares of the total spreads of the predicted outputs, are discussed.

In the work (Chang & Lee, 1994a,b), solutions have been proposed for various cases of fuzzy parameters based on the Tanaka's approach for fuzzy regression. In later years, various other approaches apart from MP-based fuzzy regression models were also discussed. Examples of these models are multi-objective (MO) approaches (Nasrabadi & Nasrabadi, 2004; Nasrabadi et al., 2005; Özelkan & Duckstein, 2000) and least square error (LSE) approaches (D'Urso & Gastaldi, 2000; Coppi et al., 2006; Chen & Hsueh, 2009).

3.2 Fuzzy least squares based approach

Another approach to the fuzzy regression problem is models that minimize the sum of squares of error between fuzzy prediction values and fuzzy observation values (Figure 4).

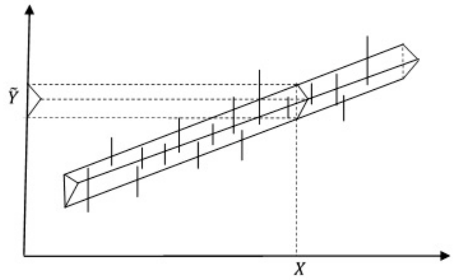

Figure 4: Fuzzy least squares model.

These models are more widely used. This approach was initiated earlier (Diamond, 1987,1988; Yen et al., 1999; D'urso, 2003). In Diamond, (1987, 1988), regression coefficients are classical numbers, but input X values are in the form of fuzzy numbers. Naturally, in the case of fuzzy input, the outputs of the predictive function will also be fuzzy numbers. The aim is to minimize the sum of squares error between the fuzzy predicted values, which are the output of the function, and the actual fuzzy observation values. Various fuzzy distance measurements can be used for this purpose (Chakraborty & Chakraborty, 2006; Nasibov, 2007; Guha & Chakraborty, 2010; Mishra et al., 2016).

In Diamond, (1987, 1988), the distance between two TFN $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$ is defined as follows:

$$D_1(A_1, A_2) = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2} \tag{12}$$

In their study (Kim & Bishu, 1998), Tanaka's and Fuzzy Least Squares (FLS) based approaches for fuzzy regression were discussed and compared in terms of membership functions. In the study (Yen et al., 1999), the regression model with coefficients of triangular fuzzy numbers was considered and solutions were produced for various shapes of TFNs. (D'urso, 2003) studied the solutions of least squares regression models in various crisp and fuzzy input/output cases.

Nasibov (2007) studied the regression model with LR-type fuzzy coefficients. The distance between fuzzy numbers is defined using the weighted average based on levels (WABL) representative of the fuzzy number and its weighted width. Based on this distance, a new minimum squares regression model is proposed. The proposed model is examined for a large class of parametric fuzzy numbers with membership degrees

$$\max(0, 1 - |x|^s), (s > 0). \tag{13}$$

The distance between LR-fuzzy numbers is defined as follows:

$$D_2(A_1, A_2) = \sqrt{(WABL(A_1) - WABL(A_2))^2 + (Width(A_1) - Width(A_2))^2} \tag{14}$$

Here, the WABL representative and weighted width of the fuzzy number are conveniently defined as follows:

$$WABL\,(A) = \int_0^1 ((1-c)\,L_A\,(\alpha) + cR_A\,(\alpha))\,p(\alpha)d\alpha, \tag{15}$$

$$Width\,(A) = \int_0^1 (R_A\,(\alpha) - L_A\,(\alpha))\,p(\alpha)d\alpha, \tag{16}$$

where, $c \in [0, 1]$, is an optimism parameter, and $p(\alpha)$ is a distribution function of the importance of the level sets that meets the following conditions:

$$\int_0^1 p(\alpha)d\alpha = 1, \tag{17}$$

$$\forall \alpha \in [0, 1] : p(\alpha) \geq 0 \tag{18}$$

In the article (Zhang & Lu, 2016), the regression model is discussed in the case of LR-type fuzzy inputs and outputs. The iterative solution of the proposed model based on the weighted least squares (WLS) estimation procedure is given. In addition, the appropriate goodness-of-fit index and its adjusted version were defined to evaluate the performance of the proposed model. Based on the WLS estimation procedure, robust estimation steps are given for the proposed model. It has been demonstrated using examples that the model reduces the effect of outliers when compared to the well-known fuzzy least squares method. In the study (Khan and Valeo, 2015), fuzzy least squares-based fuzzy linear regression model was proposed to estimate dissolved oxygen using abiotic factors in a river environment in Calgary, Canada.

3.3 Switching regression based approach

The switching regression model, also known as the Fuzzy c-Regression Model (FcRM), was first proposed in the study (Hathaway & Bezdek, 1993). There are applications of this approach in many different fields (Tezel et al., 2017; Shi, 2022). This approach is a combination of the fuzzy c-means (FCM) clustering algorithm and the regression models. In this model, it is aimed to find a regression function in the form of

$$y = f_i(\boldsymbol{x}, \boldsymbol{\beta_i}) + \varepsilon_i, i = 1, .., c, \tag{19}$$

that gives least square errors. We can express this regression model as Takagi-Sugeno Fuzzy Inference System (FIS), which consists of the following rules (Figure 5):

$$Rule\ \boldsymbol{i}:\ if\ \boldsymbol{x} \in A_i\ then\ y = f_i(\boldsymbol{x}, \boldsymbol{\beta_i}),\ i = 1, .., m. \tag{20}$$

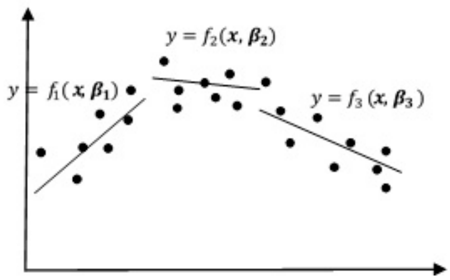

Figure 5: Switching regression model.

An iterative algorithm as follows is used to calculate A_i sets and $f_i(\boldsymbol{x}, \boldsymbol{\beta_i})$ functions in the formula (20).

FcRM Algorithm:
Initialization steps:
Step 1. Let are given: $S = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ dataset; $q > 1$, membership power parameter; For each i, let be given an error function $E = \{E_{ik}\}$ satisfying the conditions

$$E_{ik}(\boldsymbol{\beta_i}) = \|f_i(\boldsymbol{x_k}, \boldsymbol{\beta_i}) - y_k\|^2, \tag{21}$$

Let be given $\varepsilon > 0$ threshold parameter.

Step 2. The initial membership matrix $U^{(0)}$ is determined, satisfying the following conditions:

$$0 \leq U_{ik} \leq 1, \forall i, k \tag{22}$$

$$0 < \sum_{k=1}^{n} U_{ik} < n, \forall i, \tag{23}$$

$$\sum_{i=1}^{c} U_{ik} = 1, \forall k. \tag{24}$$

Iteration steps:
The following steps are repeated for iterations $r = 1, 2, \ldots,$:
Step 3. Where $f_i\left(\boldsymbol{x_k}, \boldsymbol{\beta_i}\right)$ functions are linear functions, the parameters $\boldsymbol{\beta_i} = \boldsymbol{\beta}_i^{(r)}$ that minimizes the sum of errors for each model $1 \leq i \leq c$ are calculated according to the following formula:

$$\boldsymbol{\beta}_i^{(r)} = \left[X^t D_i X\right]^{-1} X^t D_i Y \tag{25}$$

In the formula (25), X is the input matrix consisting of k.th row $\boldsymbol{x_k}$, and Y is the column vector consisting of y_k outputs. D_i is a square matrix with diagonal elements $\left(U_{ik}^{(r)}\right)^q$ membership degrees.
Step 4. Let be $E_{ik} = E_{ik}(\boldsymbol{\beta}_i^{(r)})$. Update the memberships matrix $U^{(r)} \rightarrow U^{(r+1)}$ as follows:

$$U_{ik} = \begin{cases} \dfrac{1}{\sum_{j=1}^{c}\left(\frac{E_{ik}}{E_{jk}}\right)^{\frac{1}{q-1}}}, \; if \; E_{ik} > 0 \; for \; 1 \leq i \leq c; \end{cases} \tag{26}$$

and

$$if \; \exists i : E_{ik} = 0 \; then \; U_{ik} = 1 \; and \; U_{jk} = 0, \; for \; j \neq i. \tag{27}$$

Step 5. If $\left\|U^{(r)} - U^{(r+1)}\right\| \leq \varepsilon$ then stop, otherwise set $r = r + 1$ and goto step 3.
End.

3.4 Fuzzy Decision Tree based approach

Another common approach to regression problems is decision tree (DT) approach. Regression models based on decision trees decompose the input space into certain regions and calculate the average observed values in these regions. The estimated value is calculated as the weighted average of these values. There are various regression applications of fuzzy decision trees in the literature. In the studies (Kantarci & Nasibov, 2017, 2018), a fuzzy decision tree was created on the linguistic data set by using the WABL defuzzification method. In the study (Mohammadiun et al., 2021), a framework was developed that includes the development of various integrated fuzzy decision tree regression (FDTR) models and model optimization to facilitate the selection of appropriate response methods for oil spill accidents in Arctic waters. In the (Xia et al., 2022), feature scanning followed by fuzzy regression tree using the Takagi-Sugeno fuzzy reasoning (TSFRT) hypothesis is proposed. In TSFRT, each leaf node is treated as a Takagi-Sugeno fuzzy inference system. Recently, gradient boosting models, which are new approaches to decision trees, have come to the fore. Although approaches such as fuzzy inputs and fuzzy partitioning are used for the gradient boosting regression (GBR) in many studies, target variables are accepted as exact values (Sanchez & Otero, 2007) proposed a boosting-based genetic algorithm method that can learn weighted fuzzy rules. Fuzzy logic was used to create the basic learners. The advantages of boosting methods when training fuzzy classifiers are that the size of the rule base is very small and learning is very fast. (Qin & Lawry, 2005) proposed a decision tree learning model in which the leaves are appropriate tag sets in the case of linguistic

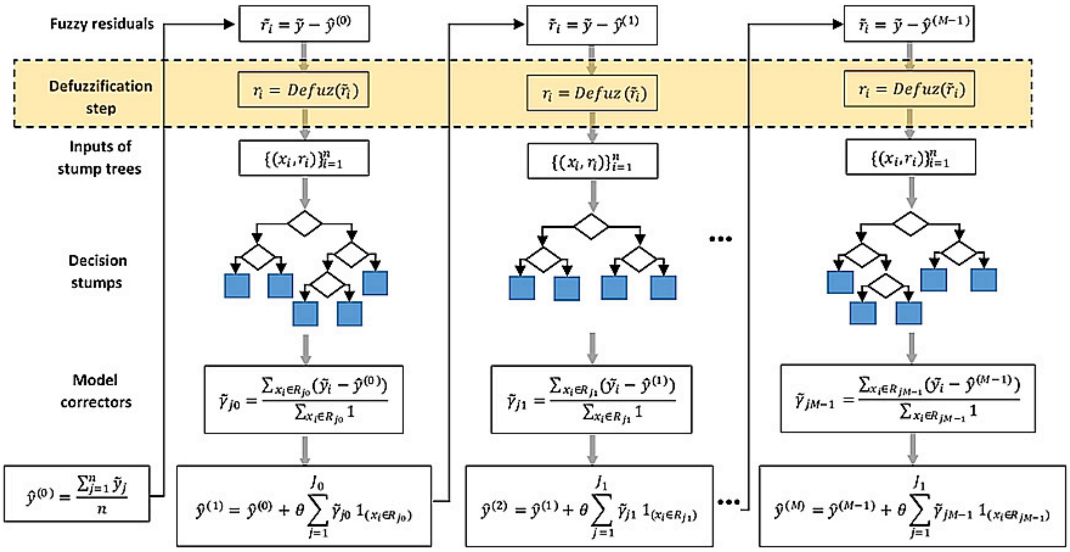

Figure 6: Graphical representation of the FuzzyGBR algorithm.

variables. In such decision trees, probability estimates for the branches in the whole tree are used instead of the majority class of the single branch in which the samples are included for classification. (Nasiboglu & Nasibov, 2022, 2023), proposed a fuzzy gradient boosting algorithm (FuzzyGBR) for cases where the observed and estimated values are in the form of triangular fuzzy numbers (Figure 6). In this papers, the following metrics are used to calculate the distances between the triangular fuzzy numbers $A = (m_1, l_1, r_1)$ and $B = (m_2, l_2, r_2)$:

$$D_3(A,\ B) \ = max(\,|m_1{-}m_2|\,,|l_1{-}l_2|\,,|r_1{-}r_2|\,), \tag{28}$$

$$D_4(A,\ B) \ = |m_1{-}m_2|+max(\,|l_1{-}l_2|\,,|r_1{-}r_2|\,), \tag{29}$$

$$D_5\,(A,\ B) \ = \ |defuz\,(A)-defuz\,(B)|\,, \tag{30}$$

Here, the $defuz()$ is any defuzzification method (Broekhoven & Baets, 2006; Nasibov & Mert, 2007; Nasibov & Shikhlinskaya, 2003; Veerraju et al., 2020; Nasiboglu & Abdullayeva, 2018; Nasibov, 2003; Nasibov, 2005; Vahidi, 2019; Mert, 2020). COA, MOM, WABL defuzzification methods are used in the study (Nasiboglu and Nasibov, 2023), and it is shown that the WABL defuzzification method is more universal and effective.

4 Analysis and conclusion

The regression problem is one of the most widely researched problems in data analysis, data mining and machine learning. Especially in the case of fuzzy information, the solution of this problem includes some different approaches compared to the classical regression problem. In this study, different approaches to the mainstream fuzzy regression models in the literature are analyzed. Among these approaches, MP based regression models and LSE based regression models are older approaches in the literature. LSE, FcRM and fuzzy decision tree based models of them are the more widely used approaches. The fuzzy gradient boosting-based approach, which is among the fuzzy decision tree models, is a relatively new approach in the literature.

The models mentioned in the study differ in terms of difficulties in processing fuzzy data, excess of running time for training algorithms, and difficulties in the case of large volumes of data. For example, there are many constraints in the mathematical programming approach when there is a lot of data, which can make it difficult to solve the optimization problem. In FcRM and fuzzy decision tree based approaches, it is better to have a lot of data and it allows to create a more robust model. On the other hand, in fuzzy LSE and fuzzy GBR based approaches,

Table 1: Effects of various factors on the models.

Models Factors	**MP based models**	**LSE based models**	**FcRM based models**	**DM based models**
Learning time	+	+	-	-
Ease of handling fuzzy data	-	-	+	-
Huge size of data volume	-	-	+	+

when many operations are performed on fuzzy data, results with very large fuzziness occur. In order to prevent this negativity, various restricted arithmetic operations should be applied on fuzzy numbers (Nasiboglu and Nasibov, 2022, 2023). In the FcRM model, the fact that there is a lot of fuzzy data does not result much negativity.

A summary of how the various factors affect the various models is given in Table 1. In the table, the "+" sign means that the factor affects the model positively or does not cause much problems, and the "-" sign means that it affects the model negatively.

In conclusion, we can state that regression models that can work with fuzzy information are widely used models in the fields of machine learning and data mining. Research on these models continues increasingly in line with new technologies developing in the field of artificial intelligence. In future studies, we think that development of software packages for the approaches based on these new technologies will be beneficial for researchers who do applied studies.

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