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NEW EXPLICIT SOLITON AND OTHER SOLUTIONS OF THE VAN DER WAALS MODEL THROUGH THE EShGEEM AND THE IEEM

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Abstract. The purpose of this note is to utilize the extended sinh–Gordon equation expansion method (EShGEEM) and the improved $\exp(-\Omega(\xi))$-expansion method (IEEM). By using the main properties of the wave transform and combine derivative, the van der Waals model equation is changed into integer-order differential equation, and the reached equation is investigated via the analytical methods. Furthermore, using the ansatz method in the form of the exp function, the symbolic computational method is used to construct kink solitary wave solutions, periodic wave solutions, as well as shock wave solutions. In addition, the physical structure and propagation characteristics of the obtained solutions are simulated. These discussions will contribute to obtain the exact solutions of nonlinear systems in which are solved by Maple software. The results presented in this note will play crucial role in these discussions and moreover the results might play an important role in the industrial applications, pharmaceutical, civil engineering and geophysics for explaining the physical meaning of the studied model. As application, an example, namely, solving the van der Waals model equation with several methods is showed.

Keywords: Extended sinh–Gordon equation expansion method, The improved $\exp(-\Omega(\xi))$-expansion method, The van der Waals model, Free parameters, Exact solutions.

AMS Subject Classification: 35Dxx, 35N05, 35G20, 35Q35, 35Q51.

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1 Introduction

The following nonlinear van der Waals equation (Barraa & Moro, 2015; Bibi et al., 2018) is considered as:

$$\phi_{tt} + (\phi_{xx} - \eta\phi_t - \phi^3 - \varepsilon\phi)_{xx} = 0, \tag{1}$$

where x and t are the variables space, time and $\phi = \phi(x,t)$ is unknown function. The dependent variable $\phi(x,t)$ is the field which reflect correction to critical average vertical density. Moreover, ε and η are the bifurcation parameter and effective viscosity respectively. A modification of the ideal gas law was proposed by Johannes D. van der Waals in 1873 to take into account molecular size and molecular interaction forces. The van der Waals model reports simple fluids in the thermodynamic limit and foretells the existence of a critical point associated to the gas-liquid phase transition. Van der Waals and ideal gas model was used to develop a Venturi flow sensor for the inspiration line to be utilized in mechanical ventilation by Politecnica (Politecnica, 2018).

In the past, a lot of potential researchers have been investigated the van der Waals equation by numerous analytical, semi-analytical and numerical techniques by the help of symbolic computations. In Herminghaus (2005) introduced and solved the van der Waals equation with the dynamic aspects of the individual liquid bridges in the sense of statistical concepts. Later on, Seadawy (2017) investigated the two-dimensional nonlinear fifth-order KP dynamical equation analytically by means of the extended auxiliary equation method and the extended modified auxiliary equation method catched some novel solutions. After the same year, Lu et al. (2017) investigated the van der Waals normal form for fluidized granular matter using the new celebrated analytical methods. Traveling wave solutions of the nonlinear evolution equations are of utmost important through the wave phenomena since they act as a bridge between mathematics and its applications in different branches of sciences (Manafian & Lakestani, 2015a; Manafian, 2015; Manafian & Lakestani, 2016a).

Due to the crucial role that exact solutions play in accurately representing the physical properties of NLPDEs in applied mathematics, For this aim, some powerful methods have been used to seek exact solutions for such equations, such as the homotopy perturbation method (Dehghan et al., 2011), fractional dirac differential operator (Shahriari & Manafian, 2020), the generalized $(G\prime/G)$-expansion method (Manafian & Allahverdiyeva, 2021), An optimal Galerkin-homotopy asymptotic method (Manafian, 2021), the stochastic data envelopment analysis (Shamsi et al., 2022), the Hirota bilinear method (Foroutan et al., 2018; Pan et al., 2022), the improved $tan(\phi/2)$-expansion method (Manafian & Lakestani, 2016c), the extended trial equation method (Manafian et al., 2017).

The main purpose of this paper is to find the solitary wave (which is sufficiently short in duration and locally irregular given in space disturbances), shock wave (it is a type of propagating disturbance that moves faster than the other waves in the medium), and singular wave (this is a type of traveling wave solutions has blow up phenomenon) solutions for the nonlinear Schrödinger equation with resonant nonlinearity the nonlinear Schrödinger equation with resonant nonlinearity (Ekici et al., 2017), Optical soliton solutions by the $tan(\phi/2)$-expansion method (Manafian, 2016), Kerr-law nonlinearity of the resonant nonlinear Schrödinger's equation (Aghdaei & Adibi, 2016), He's semi-inverse variational method to the resonant nonlinear Schrödinger's equation (Aghdaei, 2017), the Biswas-Milovic equation for Kerr law nonlinearity, the Tzitzéica type nonlinear evolution equations (Manafian & Lakestani, 2016b) and the generalized Fitzhugh-Nagumo equation with time-dependent coefficients (Manafian & Lakestani, 2015b). For discovering the exact solutions of NLPDEs one can see in the known references of the literature such as the improved $tan(\phi/2)$-expansion method for solving the sixth-order thin-film equation (Manafian et al., 2016), traveling wave solutions to the resonant Davey-Stewartson equation (Aghdaei & Manafian, 2016), kink and periodic solutions to the Kundu-Eckhaus equation (Manafian & Lakestani, 2015c), the system of equations for the ion sound and Langmuir waves in plasma (Manafian, 2017), a generalized fractional complex transform to the time fractional biological population model (Manafian & Lakestani, 2017), periodic wave solutions for Burgers, Fisher, Huxley equations (Manafian & Lakestani, 2015d).

In this study, The primary goal of this research is to present a summary of current work in identifying novel soliton solutions to the van der Waals model equation. The space-time fractional perturbed nonlinear Schrödinger equation under the Kerr law nonlinearity by using the extended sinh-Gordon equation expansion method (Sulaiman et al., 2020).

The rest of this paper is organized as follows: In the first section, the introduction is given; in the second, third, and fourth sections, we found algorithm of the EShGEEM, an improved $\exp(-\Omega(\xi))$-expansion method with a symbolic computation approach, and soliton solutions respectively. In fifth section, the physical significance and interpretation of solutions are given. In Section 6, the conclusions have been drawn.

2 Algorithm of the EShGEEM

Take the sinh–Gordon equation in the following:

$$\phi_{xt} = \alpha \sinh(\phi), \tag{2}$$

in which ϕ=$\phi(x,t)$ and α is a parameter. By using the relations $\phi(x,t)$ =$\Phi(\zeta)$ and $\zeta = kx - \omega t$, then Eq. (2) changes to the below NODE:

$$\Phi'' = -\frac{\alpha}{k\omega}\sinh(\Phi). \tag{3}$$

By integrating Eq. (3) can get the below NODE

$$\left[\left(\frac{\Phi}{2}\right)'\right]^2 = -\frac{\alpha}{k\omega}\sinh^2\left(\frac{\Phi}{2}\right) + p, \tag{4}$$

where p is a constant.
Inserting $\frac{\Phi}{2} = s(\zeta)$, and $-\frac{\alpha}{k\omega} = q$ in Eq. (4), one get

$$s' = \sqrt{p + q\sinh^2(s)}, \tag{5}$$

by selecting the parameters p and q in Eq. (5) we have the following results:
Case-I: Consider $p = 0$ and $q = 1$, hence Eq. (5) becomes

$$s' = \sinh(s). \tag{6}$$

Simplifying Eq. (6), the below findings (Manafian et al., 2016) are listed as:

$$\sinh(s) = \pm i\,\mathrm{sech}(\zeta), \cosh(s) = -\tanh(\zeta) \tag{7}$$

and

$$\sinh(s) = \pm\mathrm{csch}(\zeta), \cosh(s) = -\coth(\zeta). \tag{8}$$

in which $i = \sqrt{-1}$.
Case-II: Consider $p = 1$ and $q = 1$, hence Eq. (5) becomes

$$s' = \cosh(s). \tag{9}$$

By employing the above computation on Eq. (9), will result:

$$\sinh(s) = \tan(\zeta), \cosh(s) = \pm\sec(\zeta) \tag{10}$$

and

$$\sinh(s) = -\cot(\zeta), \cosh(s) = \pm\csc(\zeta). \tag{11}$$

Assume the following PDE

$$F(\phi, \phi_t, \phi_x, \phi_{tt}, \phi_{xx}, \phi_{xt}, \dots) = 0, \ t > 0, \tag{12}$$

in which

$$\phi(x,t) = \Phi(\zeta) \ \ \zeta = kx - \omega t. \tag{13}$$

By the help of the (13) and (12), the NODE will be found as:

$$G(\Phi, \ -\omega\Phi', \ k\Phi', \ \omega^2\Phi'', \ k^2\Phi'', ...) = 0, \tag{14}$$

where G is a polynomial of $\Phi = \Phi(\zeta)$ and its derivatives with respect to ζ. The exact solution can be considered as

$$\Phi(s) = \sum_{j=1}^{N} \cosh^{j-1}(s)\left[B_j \sinh(s) + A_j \cosh(s)\right] + A_0. \tag{15}$$

Base on the relations (6)-(8), then (15) become

$$\Phi(\zeta) = \sum_{j=1}^{N} \left(-\tanh(\zeta)\right)^{j-1}\left[\pm i B_j \operatorname{sech}(\zeta) - A_j \tanh(\zeta)\right] + A_0, \tag{16}$$

and

$$\Phi(\zeta) = \sum_{j=1}^{N} \left(-\coth(\zeta)\right)^{j-1}\left[\pm B_j \operatorname{csch}(\zeta) - A_j \coth(\zeta)\right] + A_0. \tag{17}$$

Similarly, base on the relations (9)-(11), then (15) become

$$\Phi(\zeta) = \sum_{j=1}^{N} \left(\pm \sec(\zeta)\right)^{j-1}\left[B_j \tan(\zeta) \pm A_j \sec(\zeta)\right] + A_0, \tag{18}$$

and

$$\Phi(\zeta) = \sum_{j=1}^{N} \left(\pm \csc(\zeta)\right)^{j-1}\left[-B_j \cot(\zeta) \pm A_j \csc(\zeta)\right] + A_0, \tag{19}$$

the N is the balance value.

3 The improved $\exp(-\Omega(\zeta))$-Expansion Method

Let us express the main steps of IEEM (Khan & Akbar, 2014; Rayhanul et al., 2015) in the following:

Step 1. Let us take into account the NPDE in the following

$$\mathcal{N}(\phi, \phi_x, \phi_t, \phi_{xx}, \phi_{tt}, \dots) = 0, \tag{20}$$

in which $\phi = \phi(x,t)$ is an unknown function, then the following ODE can be expressed as

$$\mathcal{Q}(\Phi, k\Phi', -\omega\Phi', k^2\Phi'', \omega^2\Phi'', ...) = 0, \tag{21}$$

in which $\zeta = kx - \omega t$, k and ω are free values.

Step 2. Suppose the following exact solution as:

$$\Phi(\zeta) = \sum_{j=-N}^{N} A_j F^j(\zeta), \tag{22}$$

where $F(\zeta) = \exp(-\Omega(\zeta))$ and $A_j(-N \le j \le N)$ such that, $A_{-N} \neq 0$, $A_N \neq 0$, and, $\Omega = \Omega(\zeta)$ satisfies the ODE as follows;

$$\Omega' = \mu F^{-1}(\zeta) + F(\zeta) + \lambda. \tag{23}$$

The exact solutions from Eq. (23) are taken as:

Set-I: If $\mu \neq 0$ and $\lambda^2 - 4\mu > 0$, then we have

$$\Omega(\zeta) = \ln\left(-\frac{\sqrt{\lambda^2 - 4\mu}}{2\mu} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\zeta + E)\right) - \frac{\lambda}{2\mu}\right). \tag{24}$$

Set-II: If $\mu \neq 0$ and $\lambda^2 - 4\mu < 0$, then we achieve

$$\Omega(\zeta) = \ln\left(\frac{\sqrt{-\lambda^2+4\mu}}{2\mu}\tan\left(\frac{\sqrt{-\lambda^2+4\mu}}{2}(\zeta+E)\right) - \frac{\lambda}{2\mu}\right). \tag{25}$$

Set-III: If $\mu = 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu > 0$, then we achieve

$$\Omega(\zeta) = -\ln\left(\frac{\lambda}{\exp(\lambda(\zeta+E)) - 1}\right). \tag{26}$$

Set-IV: If $\mu \neq 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu = 0$, then we obtain

$$\Omega(\zeta) = \ln\left(-\frac{2\lambda(\zeta+E)+4}{\lambda^2(\zeta+E)}\right). \tag{27}$$

Set-V: If $\mu = 0$, $\lambda = 0$, and $\lambda^2 - 4\mu = 0$, then we catch

$$\Omega(\zeta) = \ln(\zeta+E), \tag{28}$$

where E is a constant and $A_i(-N \leq i \leq N)$, λ and μ are values to be found. The N is found by the homogeneous balance principle.
Step 3. Putting new solution from Eq. (22) into Eq. (21) along with Eq. (23) and solving the algebraic equations including coefficients of $A_{-N}, ..., A_0, ..., A_N$, k, ω, λ, and μ into (22) we achieve to exact solution of taken problem.

4 Soliton and other solutions of the van der Waals model

To obtain the exact solution, the following relations will be considered as

$$\phi(x,t) = \Phi(\zeta), \ \zeta = kx - \omega t. \tag{29}$$

In Eq. (29), k, and ω represent the amplitude and the speed of the wave respectively. Plugging Eq. (29) into Eq. (1) we gain

$$\omega^2\Phi'' + k^2(k^2\Phi'' + \omega\eta\Phi' - \Phi^3 - \varepsilon\Phi)'' = 0. \tag{30}$$

Integration of Eq. (30) twice reduces to

$$\omega^2\Phi + k^2(k^2\Phi'' + \omega\eta\Phi' - \Phi^3 - \varepsilon\Phi) = 0, \tag{31}$$

or

$$(\omega^2 - k^2\varepsilon)\Phi + k^4\Phi'' + k^2\omega\eta\Phi' - k^2\Phi^3 = 0. \tag{32}$$

Via balancing Φ'' with Φ^3 we gives $N = 1$. In the below, we will bring two analytical methods for test the aforementioned methods in the above section.

4.1 The EShGEEM

4.1.1 For Case-I: Eq. (6)

According to the Eqs. (15)-(17), the solution of Eq. (32) can be shown as

$$\Phi(\zeta) = \pm iB_1\text{sech}(\zeta) - A_1\tanh(\zeta) + A_0, \tag{33}$$

and

$$\Phi(\zeta) = \pm B_1 \mathrm{csch}(\zeta) - A_1 \coth(\zeta) + A_0, \tag{34}$$

and so

$$\Phi(w) = B_1 \sinh(s) + A_1 \cosh(s) + A_0, \tag{35}$$

where $A_1 \neq 0$ or $B_1 \neq 0$.
Putting (35) and its derivatives into Eq. (32), the following results are obtained as:
Set 1-1: $A_0 = \sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta$, $A_1 = \frac{\eta}{2}\sqrt{\frac{2\varepsilon}{9-2\eta^2}}$, $B_1 = 0$, $k = \pm\frac{\eta}{2}\sqrt{\frac{\varepsilon}{9-2\eta^2}}$, and $\omega = \frac{3\eta\varepsilon}{18-4\eta^2}$.
By considering the above cases the dark and singular soliton solutions for the van der Waals model will be as

$$\phi_{1,2}(x,t) = \sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta - \frac{\eta}{2}\sqrt{\frac{2\varepsilon}{9-2\eta^2}} \tanh\left(\pm\frac{\eta}{2}\sqrt{\frac{\varepsilon}{9-2\eta^2}}x - \frac{3\eta\varepsilon}{18-4\eta^2}t\right), \tag{36}$$

and

$$\phi_{3,4}(x,t) = \sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta - \frac{\eta}{2}\sqrt{\frac{2\varepsilon}{9-2\eta^2}} \coth\left(\pm\frac{\eta}{2}\sqrt{\frac{\varepsilon}{9-2\eta^2}}x - \frac{3\eta\varepsilon}{18-4\eta^2}t\right). \tag{37}$$

Set 1-2: $A_0 = \sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta$, $A_1 = \sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta$, $B_1 = \pm\sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta$, $k = \pm\sqrt{\frac{\varepsilon}{9-2\eta^2}}\eta$, and $\omega = \frac{3\eta\varepsilon}{9-2\eta^2}$.
Therefore, we derive the following combined dark-bright (complexiton soliton) soliton and combined singular soliton solutions for the van der Waals model respectively:

$$\begin{aligned}\phi_{5,6}(x,t) = &\sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta - \sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta \tanh\left(\pm\sqrt{\frac{\varepsilon}{9-2\eta^2}}\eta x - \frac{3\eta\varepsilon}{9-2\eta^2}t\right) \\ &\pm\sqrt{\frac{-\varepsilon}{18-4\eta^2}}\eta \,\mathrm{sech}\left(\pm\sqrt{\frac{\varepsilon}{9-2\eta^2}}\eta x - \frac{3\eta\varepsilon}{9-2\eta^2}t\right),\end{aligned} \tag{38}$$

and

$$\begin{aligned}\phi_{7,8}(x,t) = &\sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta - \sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta \coth\left(\pm\sqrt{\frac{\varepsilon}{9-2\eta^2}}\eta x - \frac{3\eta\varepsilon}{9-2\eta^2}t\right) \\ &\pm\sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta \,\mathrm{csch}\left(\pm\sqrt{\frac{\varepsilon}{9-2\eta^2}}\eta x - \frac{3\eta\varepsilon}{9-2\eta^2}t\right).\end{aligned} \tag{39}$$

4.1.2 For Case-II: Eq. (6)

According to the Eqs. (18)-(19), the solution of Eq. (32) can be shown as

$$V(\zeta) = B_1 \tan(\zeta) \pm A_1 \sec(\zeta) + A_0, \tag{40}$$

and

$$V(\zeta) = -B_1 \cot(\zeta) \pm A_1 \csc(\zeta) + A_0, \tag{41}$$

and so

$$V(s) = B_1 \sinh(s) + A_1 \cosh(s) + A_0, \tag{42}$$

where $A_1 \neq 0$ or $B_1 \neq 0$.
Putting (42) and its derivatives into Eq. (32), the following results are obtained as:
Set 1-1: $A_0 = \sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta$, $A_1 = 0$, $B_1 = \sqrt{\frac{-\varepsilon}{18-4\eta^2}}\eta$, $k = \pm\frac{\eta}{2}\sqrt{\frac{\varepsilon}{2\eta^2-9}}$, and $\omega = \frac{3\eta\varepsilon}{18-4\eta^2}$.
The bright and singular periodic soliton solutions for the van der Waals model can be delivered as form:

$$\phi_{9,10}(x,t) = \sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta + \sqrt{\frac{-\varepsilon}{18-4\eta^2}}\eta \tan\left(\pm\frac{\eta}{2}\sqrt{\frac{\varepsilon}{2\eta^2-9}}x - \frac{3\eta\varepsilon}{18-4\eta^2}t\right), \tag{43}$$

and

$$\phi_{11,12}(x,t) = \sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta + \sqrt{\frac{-\varepsilon}{18-4\eta^2}}\eta\cot\left(\pm\frac{\eta}{2}\sqrt{\frac{\varepsilon}{2\eta^2-9}}x - \frac{3\eta\varepsilon}{18-4\eta^2}t\right). \tag{44}$$

Set 1-2: $A_0 = \sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta$, $A_1 = \pm\sqrt{\frac{-\varepsilon}{18-4\eta^2}}\eta$, $B_1 = \sqrt{\frac{-\varepsilon}{18-4\eta^2}}\eta$, $k = \pm\sqrt{\frac{\varepsilon}{9-2\eta^2}}\eta$, and $\omega = \frac{3\eta\varepsilon}{2\eta^2-9}$. Therefore, we derive the following combined singular periodic soliton solutions for the van der Waals model respectively:

$$\begin{aligned}\phi_{13,14}(x,t) = {}& \sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta + \sqrt{\frac{-\varepsilon}{18-4\eta^2}}\eta\tan\left(\pm\sqrt{\frac{\varepsilon}{9-2\eta^2}}\eta x - \frac{3\eta\varepsilon}{2\eta^2-9}t\right) \\ & \pm\sqrt{\frac{-\varepsilon}{18-4\eta^2}}\eta\sec\left(\pm\sqrt{\frac{\varepsilon}{9-2\eta^2}}\eta x - \frac{3\eta\varepsilon}{2\eta^2-9}t\right),\end{aligned} \tag{45}$$

and

$$\begin{aligned}\phi_{15,16}(x,t) = {}& \sqrt{\frac{\varepsilon}{18-4\eta^2}}\eta - \sqrt{\frac{-\varepsilon}{18-4\eta^2}}\eta\cot\left(\pm\sqrt{\frac{\varepsilon}{9-2\eta^2}}\eta x - \frac{3\eta\varepsilon}{2\eta^2-9}t\right) \\ & \pm\sqrt{\frac{-\varepsilon}{18-4\eta^2}}\eta\csc\left(\pm\sqrt{\frac{\varepsilon}{9-2\eta^2}}\eta x - \frac{3\eta\varepsilon}{2\eta^2-9}t\right).\end{aligned} \tag{46}$$

4.2 The IEEM

By considering IEEM for Eq. (32) and by balancing Φ^3 and Φ'' in Eq. (32) we can acquire the balance number $M = 1$, then the exact solution gets,

$$\Phi(\zeta) = \sum_{j=-1}^{1} A_j F^j(\zeta), \qquad F(\zeta) = \exp(-\Omega(\zeta)). \tag{47}$$

Putting (47) into Eq. (32) and comparing the terms, and by solving a system of nonlinear algebraic equations the following new results can be yielded as
Set 1: $k = \eta\sqrt{\frac{\varepsilon}{(9-2\eta^2)(\lambda^2-4\mu)}}$, $\omega = 3\eta\left[\frac{\lambda^2-8\varepsilon\mu}{2}\sqrt{\frac{2\varepsilon}{(9-2\eta^2)(\lambda^2-4\mu)}} - \frac{\lambda\varepsilon}{(9-2\eta^2)(\lambda^2-4\mu)}\right]$, $A_0 = \frac{(\lambda^2-8\varepsilon\mu)\eta}{2}$, $A_1 = 0$, and $A_{-1} = \mu\eta\sqrt{\frac{2\varepsilon}{(9-2\eta^2)(\lambda^2-4\mu)}}$.

Case I: By using of the (24) we get the following dark soliton as

$$\begin{aligned}\phi(x,t) = {}& \frac{(\lambda^2-8\varepsilon\mu)\eta}{2} - \eta\sqrt{\frac{2\varepsilon}{(9-2\eta^2)(\lambda^2-4\mu)}} \\ & \times\left\{\frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\left(\frac{\eta}{2}\sqrt{\frac{\varepsilon}{9-2\eta^2}}x - 3\eta\left[\frac{\lambda^2-8\varepsilon\mu}{4}\sqrt{\frac{2\varepsilon}{9-2\eta^2}} - \frac{\lambda\varepsilon}{2(9-2\eta^2)\sqrt{\lambda^2-4\mu}}\right]t + E\right) + \frac{\lambda}{2}\right\}.\end{aligned} \tag{48}$$

Case II: By using of the (25) we have the following periodic wave soliton as

$$\begin{aligned}\phi(x,t) = {}& \frac{(\lambda^2-8\varepsilon\mu)\eta}{2} - \eta\sqrt{\frac{2\varepsilon}{(9-2\eta^2)(\lambda^2-4\mu)}} \\ & \times\left\{-\frac{\sqrt{4\mu-\lambda^2}}{2}\tan\left(\frac{\eta}{2}\sqrt{\frac{-\varepsilon}{9-2\eta^2}}x - 3\eta\left[\frac{\lambda^2-8\varepsilon\mu}{4}\sqrt{\frac{-2\varepsilon}{9-2\eta^2}} + \frac{\lambda\varepsilon}{2(9-2\eta^2)\sqrt{4\mu-\lambda^2}}\right]t + E\right) + \frac{\lambda}{2}\right\}.\end{aligned} \tag{49}$$

Set 2: $k = \eta\sqrt{\frac{\varepsilon}{(9-2\eta^2)(\lambda^2-4\mu)}}$, $\omega = -3\eta\left[\frac{\lambda^2-8\varepsilon\mu}{2}\sqrt{\frac{2\varepsilon}{(9-2\eta^2)(\lambda^2-4\mu)}} - \frac{\lambda\varepsilon}{(9-2\eta^2)(\lambda^2-4\mu)}\right]$, $A_0 = \frac{(\lambda^2-8\varepsilon\mu)\eta}{2}$,

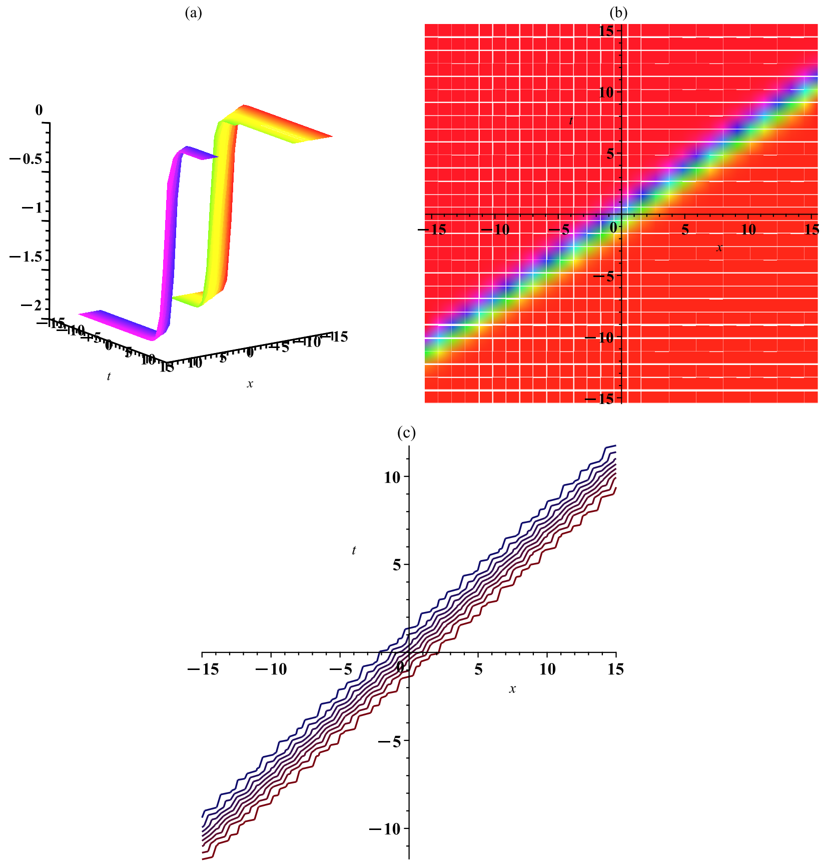

Figure 1: Simulation of Eq. (36) when (a) 3D graph, (b) density graph, and (c) contour graph.

$A_{-1} = 0$, and $A_1 = \eta\sqrt{\frac{2\varepsilon}{(9-2\eta^2)(\lambda^2-4\mu)}}$.

Case I: By using of the (24) we get the following dark-singular soliton as

$$\phi(x,t) = \frac{(\lambda^2-8\varepsilon\mu)\eta}{2} - \eta\sqrt{\frac{2\varepsilon}{(9-2\eta^2)(\lambda^2-4\mu)}} \times \left\{\frac{\sqrt{\lambda^2-4\mu}}{2\mu}\tanh\left(\frac{\eta}{2}\sqrt{\frac{\varepsilon}{9-2\eta^2}}x + 3\eta\left[\frac{\lambda^2-8\varepsilon\mu}{4}\sqrt{\frac{2\varepsilon}{9-2\eta^2}} - \frac{\lambda\varepsilon}{2(9-2\eta^2)\sqrt{\lambda^2-4\mu}}\right]t + E\right) + \frac{\lambda}{2\mu}\right\}^{-1}. \tag{50}$$

Case II: By using of the (25) we the periodic-singular soliton wave solution as

$$\phi(x,t) = \frac{(\lambda^2-8\varepsilon\mu)\eta}{2} - \eta\sqrt{\frac{2\varepsilon}{(9-2\eta^2)(\lambda^2-4\mu)}} \times \left\{-\frac{\sqrt{4\mu-\lambda^2}}{2\mu}\tan\left(\frac{\eta}{2}\sqrt{\frac{-\varepsilon}{9-2\eta^2}}x + 3\eta\left[\frac{\lambda^2-8\varepsilon\mu}{4}\sqrt{\frac{-2\varepsilon}{9-2\eta^2}} + \frac{\lambda\varepsilon}{2(9-2\eta^2)\sqrt{4\mu-\lambda^2}}\right]t + E\right) + \frac{\lambda}{2\mu}\right\}^{-1}. \tag{51}$$

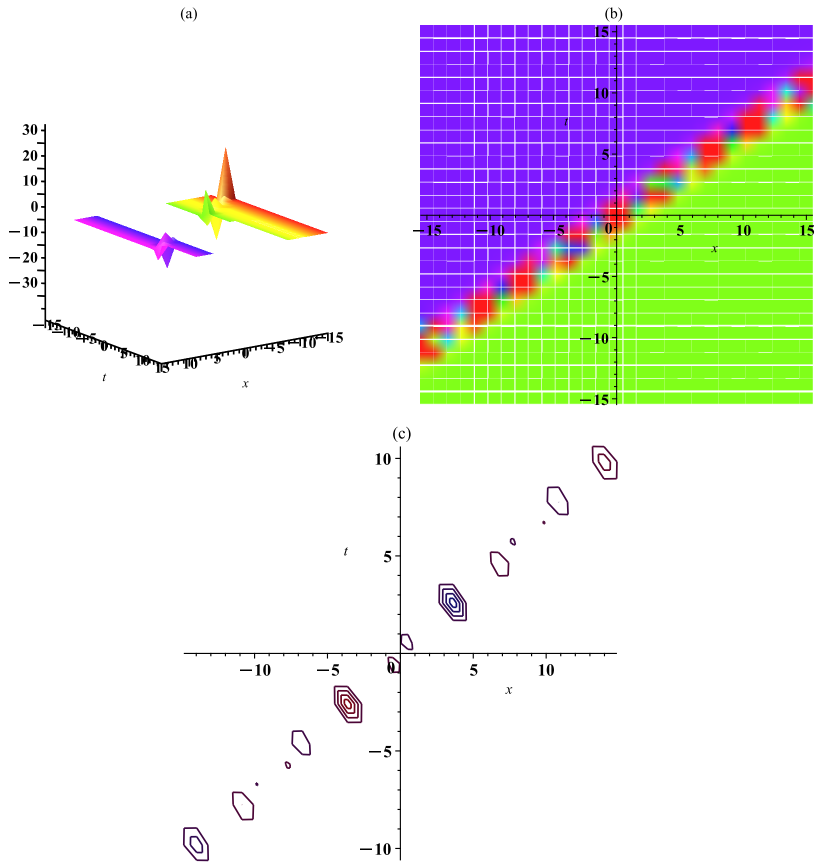

Figure 2: Simulation of Eq. (46) when (a) 3D graph, (b) density graph, and (c) contour graph.

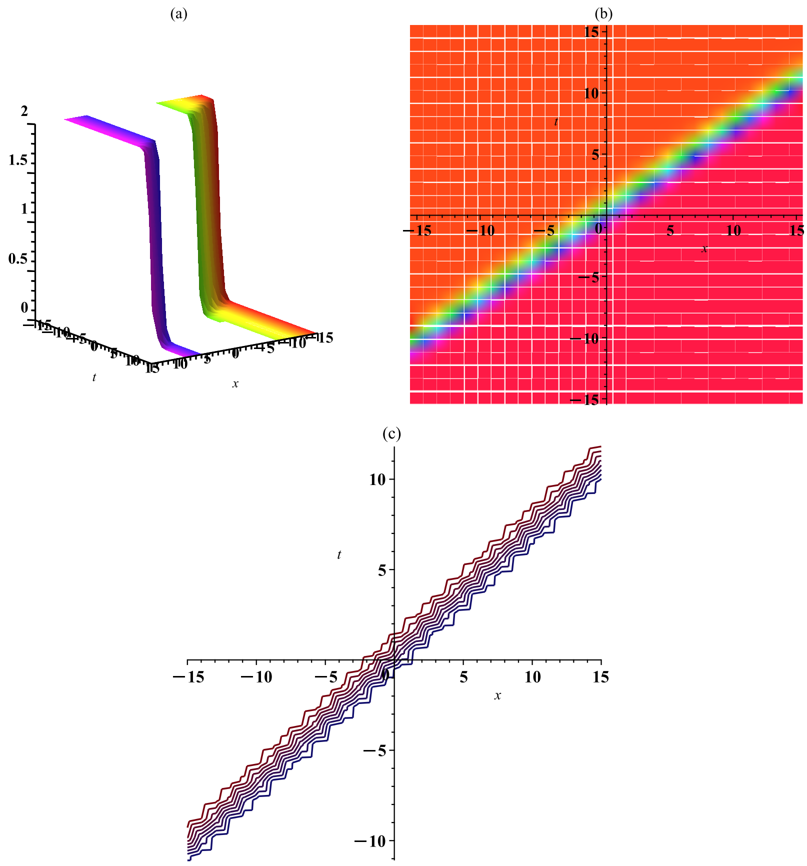

Figure 3: Simulation of Eq. (45) when (a) 3D graph, (b) density graph, and (c) contour graph.

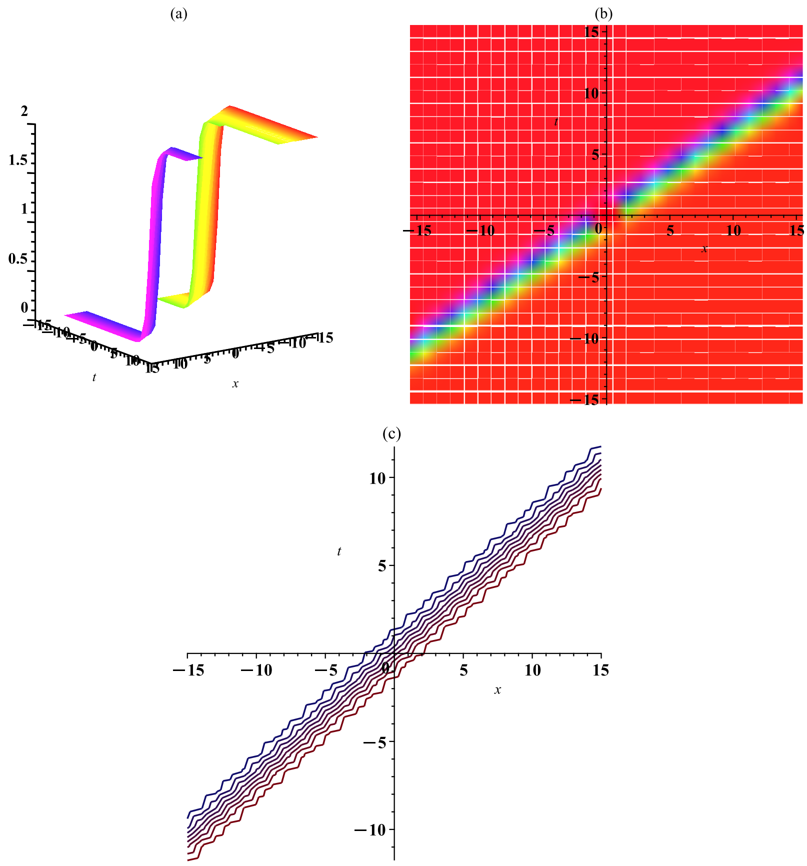

Figure 4: Simulation of Eq. (39) when (a) 3D graph, (b) density graph, and (c) contour graph.

5 Physical results of the Solutions

This section explains some of the above-constructed solutions through some distinct graphs' type (two-dimensional, three-dimensional, density and contour plots).
Figure 1: Dark soliton solution for the van der Waals model of (36) with $\varepsilon = -2$ and $\eta = -3$.
Figure 2: Singular soliton solution for the van der Waals model of (46) with $\varepsilon = -2$ and $\eta = -3$.
Figure 3: Combined dark-bright soliton solution for the van der Waals model of (45) with $\varepsilon = -2$ and $\eta = -3$.
Figure 4: Combined singular soliton solution for the van der Waals model of (39) with $\varepsilon = -2$ and $\eta = 3$. It must be noted that the results of the current paper are the outcome of theoretical modeling. There are quite a few results from experimental standpoint in mode-locked lasers that have been reported earlier. In future these analytical results will be aligned with the experimental results and the outcome of such studies will be addressed with time.

6 Conclusion

In this study, the van der Waals model equation by using the EShGEEM scheme and the IEEM scheme as the symbolic computational methods was investigated. As a result, the soliton solutions, kink-wave solutions, periodic wave solutions, dark-bright soliton solution, and shock wave solutions were found. We investigated the dynamic behavior of the obtained solutions by assigning appropriate values to the free-involved parameters. Figures 1-4 were shown the effects of x and t on soliton solutions and king solutions. The achieved analytical solitons were also explained graphically by 2-dimensional, 3-dimensional, density, and contour plots. Finally, it was suggested, to deal the other non-linear PDEs, the exp function and extended sinh-Gordon equation expansion schemes are very helpful, reliable and straight forward. Results achieved in this paper may useful for the progress in the supplementary analyzing of this model. Therefore, the reached results express that the implemented methods arising in industrial applications, pharmaceutical, civil engineering and geophysics. We drawn the graph of soliton and dark-bright soliton solutions as demonstrated in Figures 1-4, respectively.

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