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A SPECIAL SOLUTION OF CONSTANT COEFFICIENTS PARTIAL DERIVATIVE EQUATIONS WITH FOURIER TRANSFORM METHOD

Murat Düz[1*]

[1]Department of Mathematics Faculty of Science Karabuk University, Karabuk, Turkey

Abstract. In this study, we apply Fourier Transform method for general n.th order constant coefficients partial differential equations. We obtained a formula for this kind equations.

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Corresponding Author: *Murat Düz, Karabuk University, Department of Mathematics Faculty of Science, 78050 Karabuk, Turkey, e-mail: mduz@karabuk.edu.tr*

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1. Introduction

Partial differential equations are used in many areas of engineering and basic sciences. For example, the heat equation, on wave equation and the Laplace equation are some of the well-known partial differential equation used in these fields. There are some methods for solution of partial differential equations. Lagrange method, undetermined coefficients method, inverse operator method are some of the methods used to find special solutions of constant coefficient partial differential equation. Apart from these methods, partial differential equations can also be solved with Adomian decomposition, differential transformation, variational iteration methods (Ayaz, 2003; Chen & Ho, 1999; Patil & Kolte, 2015). These equations also can be solved with aid of integral transforms.

Integral transform is a mapping from functions to functions that takes the following form

$$g(\alpha) = \int_a^b f(t).K(\alpha,t)dt\,, \qquad (1)$$

where $K(\alpha,t)$ is called the kernel of transform. In (1), $f(t)$ is input fuction and $g(\alpha)$ is output function. Integral transforms have been used in solving many problems in applied mathematics, mathematical physics and engineering sciences. The best two known integral transforms are the Laplace transform and Fourier transform. The Fourier Transform, one of the gifts of Jean-Baptiste Joseph Fourier to the world of science, is an integral transform used in many areas of engineering. It has been very useful for analyzing harmonic signals or signals which are known need for local information. Moreover the Fourier transform analysis has also been very useful in many other areas such as quantum mechanics, wave motion etc. (Andrews & Shivamoggi, 1988; Bracewell, 1965; Bogges & Narcowich, 2009; Denbath & Bhattad, 2015; Murray 1974). Furthermore it has been useful in mathematics such as generalized integrals, integral equations, ordinary differential equations, partial differential equations can be solved by using the Fourier transform (Düz, 2018). Sums of some infinite series can be calculated by using Fourier series. For example value of $\int_0^\infty \frac{sinx}{x}dx$with Fourier transform or sum of $\sum_{n=1}^{\infty}\frac{1}{n^2}$ which is infinite series with Fourier series can be calculated.

Another example of its applications could be that: voice of every human can be expressed as the sum of sine and cosine. Since the electromagnetic spectrum of the frequency of each voice is different, the frequency of each sine and cosine sum will be different. In this way, a voice record can be found belongs to whom using the Fourier transform. In fact, our ear automatically runs this process instead of us (Düz, 2018).

In this study we get a formula for a special solution of *n*-th order partial derivative equations which have constant coefficients with the aid Fourier transform. The validity of the obtained formula has been seen by examples.

2. Basic Definition and Theorems

Definition 1. Fourier transform of $f(t)$

$$\mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t).e^{-iwt}dt \tag{2}$$

is defined. Since integral (2) is function of w

$$\mathcal{F}[f(t)] = F(w)$$

is written.

Definition 2. If $\mathcal{F}[f(t)] = F(w)$ then $f(t)$ is called inverse Fourier transform of $F(w)$; where

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w).e^{iwt}dt$$

and it is showed by $f(t) = \mathcal{F}^{-1}[F(w)]$.

Theorem 1. The Fourier Transform is linear,Let $c_1, c_2 \in R$. Then

$$\mathcal{F}[c_1.f_1(t) + c_2.f_2(t)] = c_1.\mathcal{F}[f_1(t)] + c_2\mathcal{F}[f_2(t)]\,.$$

Theorem 2. (Andrews and Shivamoggi, 1988) Let $f(t)$ be continuous or partly continuous in the interval $(-\infty,\infty)$ and $f(t), f'(t), f''(t), \dots, f^{(n-1)}(t) \to 0$ for $|t| \to \infty$. If $f(t), f'(t), f''(t), \dots, f^{(n-1)}(t)$ are absolutely integrable in the interval $(-\infty,\infty)$, then

$$\mathcal{F}\left[f^{(n)}(t)\right] = (iw)^n F(w).$$

Definition 3. The Dirac delta function can be rigorously thought of as a function on real line which is zero everywhere except at the origin, where it is infinite,

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}.$$

The Dirac delta function has some properties (Bracewell, 1965), that

$$\int_{-\infty}^{\infty} \delta(t)dt = 1,$$

$$\int_{-\infty}^{\infty} f(t).\delta(t - t_0)dt = f(t_0),$$

$$\int_{-\infty}^{\infty} f(t).\delta^{(n)}(t - t_0)dt = (-1)^n.f^{(n)}(t_0).$$

Theorem 3. (Andrews & Shivamoggi, 1988; Bogges & Narcowich, 2009; Bracewell, 1965) The Fourier transform of the Dirac Delta function is 1.That is $\mathcal{F}[\delta(t)] = 1$.

Theorem 4. (Andrews & Shivamoggi, 1988; Bogges & Narcowich, 2009**)**

$$(w-w_0)^n.\delta^{(n)}(w-w_0) = (-1)^n.n!\,\delta(w-w_0). \quad (3)$$

Here $\delta(w-w_0)$ is defined as following

$$\delta(w-w_0) = \begin{Bmatrix} 0, & w \neq w_0 \\ \infty, & w = w_0 \end{Bmatrix}.$$

Theorem 5. (Andrews & Shivamoggi, 1988; Bogges & Narcowich, 2009)

$$\int_{-\infty}^{\infty} \frac{\delta(w-w_0)f(w)}{(w-w_0)^n}dw = \frac{1}{n!}\frac{d^n f(w)}{dw^n}(w=w_0)$$

Proof. From Theorem 4 we know that:

$$(w-w_0)^n.\delta^{(n)}(w-w_0) = (-1)^n.n!\,\delta(w-w_0)\,.$$

Therefore we get

$$\int_{-\infty}^{\infty} \frac{\delta(w-w_0)f(w)}{(w-w_0)^n}dw = \frac{(-1)^n}{n!}\int_{-\infty}^{\infty}\delta^{(n)}(w-w_0)f(w)dw = \frac{1}{n!}\frac{d^n f(w)}{dw^n}(w=w_0)\,.$$

Theorem 6. Andrews &Shivamoggi, 1988; Bogges & Narcowich, 2009; Bracewell 1965) Fourier transforms of some functions are following

$i)\mathcal{F}[1] = 2\pi.\delta(w)$;

$ii)\ \mathcal{F}[t^n] = 2\pi.i^n.\delta^{(n)}(w)$;

$iii)\ \mathcal{F}\left[y^{(n)}\right] = (iw)^n.\mathcal{F}[y]$;

$iv)\mathcal{F}[t^n.y] = i^n\dfrac{d^n\mathcal{F}[y]}{dw^n}$;

$v)\mathcal{F}[e^{at}] = 2\pi\delta(w+ia)$;

$vi) If\ \mathcal{F}[y] = Y(w), than\ \mathcal{F}[e^{w_0.t}.f(t)] = Y(w+iw_0)$.

Lemma 1. Let $f(x,y)$ be continuous or partly continuous in the interval $(-\infty,\infty)$ and $f(x,y), \frac{\partial}{\partial x}f(x,y), \frac{\partial^2}{\partial x^2}f(x,y), \dots, \frac{\partial^{n-1}}{\partial x^{n-1}}f(x,y) \to 0$ for $|x| \to \infty$.

If $f(x,y), \frac{\partial}{\partial x}f(x,y), \frac{\partial^2}{\partial x^2}f(x,y), \dots, \frac{\partial^{n-1}}{\partial x^{n-1}}f(x,y)$ are absolutely integrable in interval$(-\infty,\infty)$, then Fourier transforms of partial derivatives n −th order of $f(x,y)$ are following.

$$i)\mathcal{F}\left[\frac{\partial^n f}{\partial y^n}\right] = \frac{\partial^n F(w,y)}{\partial y^n},$$

$$ii)\ \mathcal{F}\left[\frac{\partial^n f}{\partial x^n}\right] = (iw)^n F(w,y)\,.$$

Proof. i) $\mathcal{F}\left[\frac{\partial^n f}{\partial y^n}\right] = \int_{-\infty}^{\infty}\frac{\partial^n f}{\partial y^n}.e^{-iwx}dx =$

$$= \frac{\partial^n}{\partial y^n} \int_{-\infty}^{\infty} f(x,y).\, e^{-iwx}\, dx = \frac{\partial^n F(w,y)}{\partial y^n}$$

ii) Let's do proof with induction. For $n = 1$

$$\mathcal{F}\left[\frac{\partial f}{\partial x}\right] = \int_{-\infty}^{\infty} \frac{\partial f}{\partial x} e^{-iwx}\, dx =$$

$$= \lim_{a\to-\infty} \lim_{b\to-\infty} e^{-iwx} f(x,y)\Big|_a^b + iw \int_{-\infty}^{\infty} f(x,y).\, e^{-iwx}\, dx = iwF(w,y)\,.$$

Thus lemma is true for $n = 1$.

We assume that lemma is true for $n = m$. Let's

$$\mathcal{F}\left[\frac{\partial^m f}{\partial x^m}\right] = (iw)^m F(w,y)\,.$$

We must show that lemma is true $n = m + 1$.

$$\mathcal{F}\left[\frac{\partial^{m+1} f}{\partial x^{m+1}}\right] = \int_{-\infty}^{\infty} \frac{\partial^{m+1} f}{\partial x^{m+1}}.\, e^{-iwx}\, dx\ .$$

If we apply partial integration method than we get that

$$\mathcal{F}\left[\frac{\partial^{m+1} f}{\partial x^{m+1}}\right] = \left[\left(\lim_{a\to-\infty} \lim_{b\to-\infty} e^{-iwx} \frac{\partial^m f}{\partial x^m}\Big|_a^b\right) + iw \int_{-\infty}^{\infty} \frac{\partial^m f}{\partial x^m}.\, e^{-iwx}\, dx\right] =$$

$$= iw\mathcal{F}\left[\frac{\partial^m f}{\partial x^m}\right] = iw.\,(iw)^m F(w,t) = (iw)^{m+1} F(w,y)\,.$$

Thus proof of ii) is completed.

Theorem 7. Fourier transforms of partial derivatives $(n+m)$.th order of $f(x,y)$are following.

$$\mathcal{F}\left[\frac{\partial^{m+n} f}{\partial x^n \partial y^m}\right] = (iw)^n \frac{\partial^m}{\partial y^m}\big(F(w,y)\big)\,. \tag{4}$$

Proof.

$$\mathcal{F}\left[\frac{\partial^{m+n} f}{\partial x^n \partial y^m}\right] = \int_{-\infty}^{\infty} \frac{\partial^{n+m} f}{\partial x^n \partial y^m}.\, e^{-iwx}\, dx =$$

$$= \frac{\partial^m}{\partial y^m} \int_{-\infty}^{\infty} \frac{\partial^n f}{\partial x^n}.\, e^{-iwx}\, dx = \frac{\partial^m}{\partial y^m}\ \mathcal{F}\left[\frac{\partial^n f}{\partial x^n}\right] =$$

$$= \frac{\partial^m}{\partial y^m}((iw)^n F(w,y)) = (iw)^n \frac{\partial^m F(w,y)}{\partial y^m}\,.$$

3. Solution of constant coefficients partial derivative equations from n-th order

Theorem 8. Let $A_{i,j}$ are real constants $(1 \le i \le n, 1 \le j \le n, i+j \le n), u = u(x,y)$ is a polynomial of x, y. Then a solution of

$$A_{n,0}\frac{\partial^n u}{\partial x^n} + A_{n-1,1}\frac{\partial^n u}{\partial x^{n-1}\partial y} + A_{n-2,2}\frac{\partial^n u}{\partial x^{n-2}\partial y^2} + \cdots + A_{0,n}\frac{\partial^n u}{\partial y^n} +$$
$$+A_{n-1,0}\frac{\partial^{n-1} u}{\partial x^{n-1}} + A_{n-2,1}\frac{\partial^{n-1} u}{\partial x^{n-2}\partial y} + A_{n-3,2}\frac{\partial^{n-1} u}{\partial x^{n-3}\partial y^2} + \cdots + A_{0,n-1}\frac{\partial^{n-1} u}{\partial y^{n-1}} +$$
$$+ \cdots + A_{1,0}\frac{\partial u}{\partial x} + A_{0,1}\frac{\partial u}{\partial y} + A_{0,0}u =$$
$$= F(x,y)$$

is

$$u = \mathcal{F}^{-1}\left[\frac{\mathcal{F}\big(F(x,y)\big)}{L(D)}\right],$$

where

$$L(D) = A_{0,n}D^n + \Big((iw)A_{1,n-1} + A_{0,n-1}\Big)D^{n-1} + \big((iw)^2A_{2,n-2} + (iw)A_{1,n-2} + A_{0,n-2}\big)D^{n-2} +$$
$$+ \cdots + \big(A_{n,0}(iw)^n + A_{n-1,0}(iw)^{n-1} + \cdots + A_{1,0}(iw) + A_{0,0}\big).$$

Proof. The equation in the theorem can been written by using sum symbol as following.

$$\sum_{k=0}^{n} A_{n-k,k}\frac{\partial^n u}{\partial x^{n-k}\partial y^k} + \sum_{k=0}^{n-1} A_{n-1-k,k}\frac{\partial^{n-1} u}{\partial x^{n-1-k}\partial y^k} + \cdots + \sum_{k=0}^{1} A_{1-k,k}\frac{\partial u}{\partial x^{1-k}\partial y^k}$$
$$+ A_{0,0}u = F(x,y).$$

We let's use Fourier transform for above equality

$$\sum_{k=0}^{n} A_{n-k,k}\mathcal{F}\left(\frac{\partial^n u}{\partial x^{n-k}\partial y^k}\right) + \sum_{k=0}^{n-1} A_{n-1-k,k}\mathcal{F}\left(\frac{\partial^{n-1} u}{\partial x^{n-1-k}\partial y^k}\right) + \cdots + A_{0,0}\mathcal{F}(u)$$
$$= \mathcal{F}\big(F(x,y)\big).$$

From theorem 7 (or equality 4) we get following equality.

$$\sum_{k=0}^{n} A_{n-k,k}(iw)^{n-k}\frac{\partial^k U}{\partial y^k} + \sum_{k=0}^{n-1} A_{n-1-k,k}(iw)^{n-1-k}\frac{\partial^k U}{\partial y^k} + \cdots$$
$$+ \sum_{k=0}^{1} A_{1-k,k}(iw)^{1-k}\frac{\partial^k U}{\partial y^k} + A_{0,0}U = \mathcal{F}\big(F(x,y)\big).$$

Here $U = U(w,y)$ is fourier transform of $u(x,y)$.

We can write above equality as following

$$A_{n,0}(iw)^n U + A_{n-1,1}(iw)^{n-1}\frac{\partial U}{\partial y} + A_{n-2,2}(iw)^{n-2}\left(\frac{\partial^2 U}{\partial y^2}\right) + \cdots + A_{0,n}\left(\frac{\partial^n U}{\partial y^n}\right) +$$

$$+A_{n-1,0}(iw)^{n-1}U + A_{n-2,1}(iw)^{n-2}\left(\frac{\partial U}{\partial y}\right) + A_{n-3,2}(iw)^{n-3}\left(\frac{\partial^2 U}{\partial y^2}\right) + \cdots$$

$$+ A_{0,n-1}\left(\frac{\partial^{n-1} U}{\partial y^{n-1}}\right) +$$

$$+ \cdots + A_{1,0}(ik)U + A_{0,1}\frac{\partial U}{\partial y} + A_{0,0}U =$$

$$= \mathcal{F}(F(x,y)).$$

$$A_{0,n}D^n U + \left((iw)A_{1,n-1} + A_{0,n-1}\right)D^{n-1}U$$
$$+ \left((iw)^2 A_{2,n-2} + (iw)A_{1,n-2} + A_{0,n-2}\right)D^{n-2}U$$

$$+ \cdots + \left(A_{n,0}(iw)^n + A_{n-1,0}(iw)^{n-1} + \cdots + A_{1,0}(iw) + A_{0,0}\right)U =$$

$$= \mathcal{F}\left(F(x,y)\right),$$

where $D^n = \frac{\partial^n}{\partial y^n}$.

$$\Big[A_{0,n}D^n + \left((iw)A_{1,n-1} + A_{0,n-1}\right)D^{n-1} + \left((iw)^2 A_{2,n-2} + (iw)A_{1,n-2} + A_{0,n-2}\right)D^{n-2}$$
$$+ \cdots + \left(A_{n,0}(iw)^n + A_{n-1,0}(iw)^{n-1} + \cdots + A_{1,0}(iw) + A_{0,0}\right)\Big]U = \mathcal{F}\left(F(x,y)\right).$$

Therefore, solution is obtained as following

$$L(D) = A_{0,n}D^n + \left((ik)A_{1,n-1} + A_{0,n-1}\right)D^{n-1} + \left((ik)^2 A_{2,n-2} + (ik)A_{1,n-2} + A_{0,n-2}\right)D^{n-2} +$$
$$+ \cdots + \left(A_{n,0}(ik)^n + A_{n-1,0}(ik)^{n-1} + \cdots + A_{1,0}(ik) + A_{0,0}\right),$$

$$U(w,y) = \frac{\mathcal{F}(F(x,y))}{L(D)},$$

$$u(x,y) = \mathcal{F}^{-1}\left(U(w,y)\right).$$

4. Conclusion

Let A, B, C be real constants, $F = F(x,y)$ be a polynomial of x, y. Then a solution of

$$A\frac{\partial u}{\partial x} + B\frac{\partial u}{\partial y} + Cu = F(x,y)$$

is

$$u(x,y) = \mathcal{F}^{-1}\left[\frac{1}{e^{(Aiw+C)y}}\int \frac{\tilde{F}(w,y)}{B} e^{(Aiw+C)y}\,dy\right],$$

where $\tilde{F}(w,y)$ is fourier transform of $F(x,y)$.

Example 1. Find a special solution of following equation.

$$u_x + 2u_y - 5u = 2e^{5x} .$$

Solution. Fourier transform of the solution of above equation from conclusion is that

$$U(x,y) = \frac{1}{e^{\frac{iw-5}{2}y}} \int \frac{2.2\pi.\delta(w+5i)}{2}.e^{\frac{iw-5}{2}y} dy =$$

$$= \frac{4\pi}{iw-5}\ \delta(w+5i).$$

Therefore

$$u(x,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{4\pi}{iw-5} \delta(w+5i).e^{iwx} dw =$$

$$= \frac{2}{i}\frac{\partial}{\partial w}\left(e^{iwx}\right)(w=-5i)=$$

$$= 2x.e^{5x} .$$

Example 2. Find a special solution of following equation

$$u_x + u_y - u = 2xe^y.$$

Solution. Fourier transform of the solution of above equation from conclusion is that

$$U(x,y) = \frac{1}{e^{(iw-1)y}} \int 4\pi i e^y \delta'(w).e^{(iw-1)y} dy =$$

$$= \frac{4\pi i}{iw} \delta'(w).e^y .$$

Therefore

$$u(x,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{4\pi}{u'} \delta'(w).e^y.e^{iwx} dw =$$

$$= 2e^y \int_{-\infty}^{\infty} -\frac{\delta(w)}{w^2}.e^{iwx} dw =$$

$$= -2e^y \frac{1}{2!}\frac{\partial^2\left(e^{iwx}\right)}{\partial w^2}(w=0).$$

$$= x^2e^y$$

Example 3. (Patil & Kolte, 2008) Find a special of following equation.

$$u_{xx} + 16u_{yy} = x^2y^2.$$

Solution . From theorem coeffients of equation are $n=2,\ A_{2,0}=1, A_{1,1}=0,\ A_{0,2}=-1, A_{1,0}=A_{0,1}=A_{0,0}=0$. Therefore we can write that:

$$U(w,y)=\frac{y^2.2\pi.i^2\delta''(w)}{-16D^2+(iw)^2}=\frac{y^2.2\pi.\delta''(w)}{16D^2+w^2}=$$

$$=2\pi.\delta''(w)\frac{1}{16D^2+w^2}(y^2)=$$

$$=2\pi.\delta''(w)\frac{1}{w^2\left(1+\frac{16D^2}{w^2}\right)}(y^2)=$$

$$=\frac{2\pi.\delta''(w)}{w^2}\left(1-\frac{16D^2}{w^2}+\frac{256D^4}{w^4}-\cdots\right)(y^2)=$$

$$=\frac{2\pi.y^2.\delta''(w)}{w^2}-\frac{64\pi\delta''(w)}{w^4}.$$

From inverse fourier transform

$$u(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2\pi y^2\delta''(w)}{w^2}.e^{iwx}dw-\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{64\pi\delta''(w)}{w^4}.e^{iwx}dw=$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\pi y^2\delta(w)}{w^4}.e^{iwx}dw-\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{128\pi\delta(w)}{w^6}.e^{iwx}dw=$$

$$=\frac{1}{2\pi}\frac{4\pi y^2}{4!}\frac{\partial^4\left(e^{iwx}\right)}{\partial w^4}(w=0)-\frac{1}{2\pi}\frac{128\pi}{6!}\frac{\partial^6\left(e^{iwx}\right)}{\partial w^6}(w=0)=$$

$$=\frac{y^2x^4}{12}+\frac{4x^6}{45}.$$

Example 4. Find a special of following equation.

$$u_{xx}-u_{xy}-2u_{yy}=(y-1)e^x.$$

Solution. From the theorem coefficients of equation are $A_{2,0}=1$, $A_{1,1}=-1$, $A_{0,2}=-2$, $A_{1,0}=A_{0,1}=A_{0,0}=0$. Therefore fourier transform of solution are that:

$$U(w,y)=\frac{\mathcal{F}[(y-1)e^x]}{(-2D^2-iwD+(iw)^2)}=$$

$$=\frac{1}{-w^2\left(1+\frac{iD}{w}+\frac{2D^2}{w^2}\right)}[(y-1).2\pi.\delta(w+i)]=$$

$$=\frac{2\pi.\delta(w+i)}{-w^2}\left(1-\frac{iD}{w}-\frac{2D^2}{w^2}+\left(\frac{iD}{w}+\frac{2D^2}{w^2}\right)^2+\cdots\right)(y-1)=$$

$$=\frac{2\pi.\delta(w+i)}{-w^2}\left(y-1-\frac{i}{w}\right)=\frac{2\pi.\delta(w+i).(y-1)}{w^2}+\frac{2\pi.i.\delta(w+i)}{w^3}.$$

$$u(x,y)=\mathcal{F}^{-1}[U(w,y)]=\mathcal{F}^{-1}\left[\frac{2\pi.\delta(w+i).(y-1)}{w^2}+\frac{2\pi.i.\delta(w+i)}{w^3}\right]=$$

$$= \int_{-\infty}^{\infty} \frac{2\pi.\delta(w+i).(y-1)}{w^2}.e^{iwx}\,dw + \int_{-\infty}^{\infty} \frac{2\pi.i.\delta(w+i)}{w^3}e^{iwx}\,dw$$

$$= (y-1)e^x + e^x = y.e^x.$$

References

Andrews, L.C., Shivamoggi, B.K. (1988). *Integral Transforms for Engineering*, SPIE Press.

Ayaz, F. (2003). On the two-dimensional differential transform method. *Applied Mathematics and Computation*, 143, 361-374.

Bildik, N., Konuralp, A., Orakçı, Beka, F., Küçükarslan, S. (2006). Solution of different type of the partial differential equation by differential transform method and Adomian's decomposition method. *Applied Mathematics and Computation*, 172(1), 551-567.

Bogges, A., Narcowich, F.J. (2009). *A first Course in Wavelets with Fourier Analysis*, 2nd Edition. Wiley.

Bracewell, R.N. (1965). *The Fourier Transfiorm and Its Applications*. Boston: McGraw-HillBook Campany.

Chen, C.K., Ho, S.H. (1999). Solving partial differential equtions by two-dimensional differential transform method. *Applied Mathematics and Computation*, 106, 171-179.

Denbath, L., Bhattad, D. (2015). *Integral Transforms and Their Applications*. CRC Press Taylor Francis Group Network.

Düz, M. (2018). Solution of complex differential equations by using Fourier transform. *International Journal of Applied Mathematics*, *31*(1), 23-32.

Murray, R.S. (1974). *Fourier Analysis.* Schaum's Outline Series, McGraw Hill.

Patil, P., Kolte, A. (2015). Solution of Partial Differential Equation by Undetermined Coefficients Method. *International Journal of Engineering and Innovative Technology, 4*(9), 93-95.