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IMPACT OF TEMPERATURE ON PROFIT ESTIMATION OF TWO FISHERMEN EXPLOITING THREE COMPETING SPECIES USING MARKOV CHAIN*

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Abstract. The species extinction is caused sometimes by environmental forces such as habitat fragmentation, climate change, natural disaster, evolutionary changes etc, and sometimes by over-exploitation by humans, and pollution, to preserve the biodiversity in order to protect the ecosystem and the environmental life cycle, it is essential to predict the probabilities of the future in the way to interfere to save and protect the species from potential extinction. As the temperature factor is an important element for marine species, in this paper, we aim to estimate the temperature factor by discrete time Markov Chain, and then estimate the profit of two fishermen exploiting three species with some numerical simulations at the end.

Keywords: Bioeconomic model, Species in competition, Fishing effort, Fishing Profit, Discrete Time Markov Chain.

AMS Subject Classification: 60J10.

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1 Introduction

It is known that competition between species ensures biodiversity and evolution, where competition is an interaction between organisms or species in which both require a resource that is in limited supply such as food, water, or territory (Begon, 1996). So, the species that compete with each other actually they ensure their evolution and breed to preserve their survival and continuation in the way to prevent extinction (Lindegren et al., 2013; Mehrim et al., 2023).

According to Raup (1994), the role of extinction in evolution is not very well understood and may depend on which type of extinction is considered may be the result of competition between species for limited resources (the competitive exclusion principle) or climate change (Kutscheran, 2004; Dainys et al., 2019).

As reported by the International Union for Conservation of Nature's 1,616 species of fish are at risk of extinction; another 989 are endangered and 627 are critically endangered. In this context, we consider fish exploitation by human as an interesting indicator of endangered fish harvested for food, as there are many aspects of danger.

In this work, we study three competing species which are mainly harvested for food.

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The three of the species are supposed to live in the Atlantic ocean (Casablanca region) which are in competition for food and space, and also suffers from the highest exploitation rate harvest for food.

This work aims to study the temperature effect on the fishermen profit, the temperature factor is considered as an external factor of the bioeconomic Model that Maximises the profit of two fishermen exploiting three species in competition, The maximisation problem is solved using the Nash Equilibrium problem which announces that actors must not deviate from their initial strategy to ensure the desired outcome that (El Foutayeni & Khaladi, 2010; 2012), and (El Foutayeni & Zegzouti, 2012) developed, yet the considered system is not a closed one, as there are external factors that can affect the desired results which temperature is one of them (Jensen et al., 2017).

The choice of the temperature factor is based on its effect on the elements of our system; its effect the fishermen side and also species side, which ensures that it affects the profit consequently (Skov & Arlinghaus, 2017).

The paper is structured as follows: The section 2, contains the hypothesis considered in the model construction. The Section 3, defines the biological model of fish populations. The section 4, defines the bioeconomic model of the fish populations taking into consideration the fact that the prices of fish populations vary according to the quantity harvested (El Foutayeni, 2012; Bentounsi et al., 2018). In The section 5, we study the profit Estimation of fishermen considering temperature factor. In section 6, we present some numerical simulations to illustrate the results, and in Section 7, we present a discussion around the results of this work. Finally, in Section 8 we give a conclusion.

2 Hypothesis

1. We suppose that the biomass of the three marine species are strictly positive.
2. We suppose that the biomass growth follows logistic growth.
3. We suppose that the predators of the three species are not taken in consideration in this model.
4. We suppose that the three species are in comepetition for food and space.
5. We suppose that pandemic problems are not considered in this model.
6. We suppose that the temperature factor follows the Markov Proprety.

3 Biological Model

The purpose of this section is to define the biological equilibrium of the three competitive marine species, and whose natural growth of each is obtained by the logistic law.

3.1 The Biomass Evolution Model of The Three Populations

The evolution of the biomass of the fish population in competition is modeled by the following mathematical system of logistic growth.

$$\begin{cases} \dot{D}_1 = r_1 D_1 \left(1 - \frac{D_1}{K_1}\right) - c_{12} D_1 D_2 - c_{13} D_1 D_3 \\ \dot{D}_2 = r_2 D_2 \left(1 - \frac{D_2}{K_2}\right) - c_{21} D_2 D_1 - c_{23} D_2 D_3 \\ \dot{D}_3 = r_3 D_3 \left(1 - \frac{D_3}{K_3}\right) - c_{31} D_1 D_3 - c_{32} D_2 D_3, \end{cases} \tag{1}$$

where $(D_j)_{j=1,2,3}$ are the densities of populations; $(r_j)_{j=1,2,3}$ are the intrinsic growth rates; $(K_j)_{j=1,2,3}$ are the carrying capacities for the respective species; and $(c_{jk})_{1\leq j\neq k\leq 3}$ are the coefficients of competition between species k and species j.

Proposition 1. *The persistent model of a dynamical system of differential equations can be estimated by setting all derivativeses to zero.*

The steady states of the system of equations (1) are obtained by solving the equations

$$\begin{cases} r_1 D_1^* \left(1 - \frac{D_1^*}{K_1}\right) - c_{12} D_1^* D_2^* - c_{13} D_1^* D_3^* = 0 \\ r_2 D_2^* \left(1 - \frac{D_2^*}{K_2}\right) - c_{21} D_2^* D_1^* - c_{23} D_2^* D_3^* = 0 \\ r_3 D_3^* \left(1 - \frac{D_3^*}{K_3}\right) - c_{31} D_1^* D_3^* - c_{32} D_2^* D_3^* = 0, \end{cases} \tag{2}$$

where $\left(D_j^*\right)_{j=1,2,3}$ are the steady states of the three populations growth.

The solution of system (2) is defined by $P(D_1^*, D_2^*, D_3^*)$ as follows

$$\begin{aligned} D_1^* = \; & \tfrac{1}{\chi} K_1 \left(r_t - K_2 r_2 r_3 c_{12} - K_3 r_2 r_3 c_{13}\right) + \\ & \tfrac{1}{\chi} K_1 \left(-K_2 K_3 r_1 c_{23} c_{32} + K_2 K_3 r_2 c_{13} c_{32} + K_2 K_3 r_3 c_{12} c_{23}\right) \\ D_2^* = \; & \tfrac{1}{\chi} K_2 \left(r_t - K_1 r_1 r_3 c_{21} - K_3 r_1 r_3 c_{23}\right) + \\ & \tfrac{1}{\chi} K_2 \left(K_1 K_3 r_1 c_{31} c_{23} - K_1 K_3 r_2 c_{13} c_{31} + K_1 K_3 r_3 c_{21} c_{13}\right) \\ D_3^* = \; & \tfrac{1}{\chi} K_3 \left(r_t - K_1 r_1 r_2 c_{31} - K_2 r_1 r_2 c_{32}\right) + \\ & \tfrac{1}{\chi} K_3 \left(K_1 K_2 r_1 c_{21} c_{32} + K_1 K_2 r_2 c_{12} c_{31} - K_1 K_2 r_3 c_{12} c_{21}\right), \end{aligned} \tag{3}$$

where $\chi = r_t + K_1 K_2 c_{12} \left(K_3 c_{31} c_{23} - r_3 c_{21}\right) + K_3 K_2 c_{32} \left(K_1 c_{21} c_{13} - r_1 c_{23} c_{32}\right) - K_1 K_3 r_2 c_{13} c_{31}$ and $r_t = r_1 r_2 r_3$.

Remark 1. *This system admits other solutions that we didn't consider as it contradicts the hypothesis; for example, the point $P(0,0,0)$ is neglected as the system will lose its meaning if the densities are equal to 0.*

Proposition 2. *A steady state is stable if it is locally asymptotically stable.*

Theorem 1. *The point $P(D_1^*, D_2^*, D_3^*)$ is locally asymptotically stable.*

Proof. We proof this theorem by using Routh-Hurwitz stability criterion.

The variational matrix of the system at the steady state $P(D_1^*, D_2^*, D_3^*)$ is

$$V = \begin{bmatrix} V_{11} & -c_{12} D_1^* & -c_{13} D_1^* \\ -c_{21} D_1^* & V_{22} & -c_{23} D_2^* \\ -c_{31} D_3^* & -c_{32} D_3^* & V_{33} \end{bmatrix}, \tag{4}$$

where

$$\begin{cases} V_{11} = r_1 (1 - \frac{2D_1^*}{K_1}) - c_{12} D_2^* - c_{13} D_3^* \\ V_{22} = r_2 (1 - \frac{2D_2^*}{K_2}) - c_{21} D_1^* - c_{23} D_3^* \\ V_{33} = r_3 (1 - \frac{2D_3^*}{K_3}) - c_{31} D_1^* - c_{32} D_2^*. \end{cases} \tag{5}$$

Using (2) we have

$$\begin{cases} r_1 D_1^* \left(1 - \frac{2D_1^*}{K_1}\right) - c_{12} D_1^* D_2^* - c_{13} D_1^* D_3^* = -\, r_1 \frac{D_1^*}{K_1} \\ r_2 D_2^* \left(1 - \frac{2D_2^*}{K_2}\right) - c_{21} D_2^* D_1^* - c_{23} D_2^* D_3^* = -r_2 \frac{D_2^*}{K_2} \\ r_3 D_3^* \left(1 - \frac{2D_3^*}{K_3}\right) - c_{31} D_1^* D_3^* - c_{32} D_2^* D_3^* = -\, r_3 \frac{D_3^*}{K_3}, \end{cases} \tag{6}$$

Then

$$V = \begin{bmatrix} -r_1\frac{D_1^*}{K_1} & -c_{12}D_1^* & -c_{13}D_1^* \\ -c_{21}D_2^* & -r_2\frac{D_2^*}{K_2} & -c_{23}D_2^* \\ -c_{31}D_3^* & -c_{32}D_3^* & -r_3\frac{D_3^*}{K_3} \end{bmatrix}. \tag{7}$$

However the biological model is meaningful only if the biomasses of the three species are strictly positive; $D_i^* > 0$.

The polynomial format of the variational matrix is represented by:

$$Q(\lambda) = a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_0 \ , \tag{8}$$

where

$$\begin{aligned} a_0 =& \ 1 \\ a_1 =& \ \tfrac{1}{K_1}r_1D_1^* + \tfrac{1}{K_2}r_2D_2^* + \tfrac{1}{K_3}r_3D_3^* \\ a_2 =& \ \tfrac{1}{K_1K_2}r_1r_2D_1^*D_2^* - c_{13}c_{31}D_1^*D_3^* - c_{23}c_{32}D_2^*D_3^* - \\ & c_{12}c_{21}D_1^*D_2^* + \tfrac{1}{K_1K_3}r_1r_3D_1^*D_3^* + \tfrac{1}{K_2K_3}r_2r_3D_2^*D_3^* \\ a_3 =& \ c_{12}c_{31}c_{23}D_1^*D_2^*D_3^* + c_{21}c_{13}c_{32}D_1^*D_2^*D_3^* - \tfrac{r_1}{K_1}c_{23}c_{32}D_1^*D_2^*D_3^* - \\ & \tfrac{r_2}{K_2}c_{13}c_{31}D_1^*D_2^*D_3^* - \tfrac{r_3}{K_3}c_{12}c_{21}D_1^*D_2^*D_3^* + \tfrac{r_1r_2r_3}{K_1K_2K_3}D_1^*D_2^*D_3^*. \end{aligned}$$

As for $r_i > c_{ij}K_j$,$\forall i, j = 0, 1, 2, 3$, with $i \neq j$, we have

$$\begin{cases} a_0 = 1 > 0 \\ a_1 = \frac{r_1}{K_1}D_1^* + \frac{r_2}{K_2}D_2^* + \frac{r_3}{K_3}D_3^* > 0 \\ a_2 > 0 \\ a_3 > 0. \end{cases} \tag{9}$$

Consequently we have:

$$\begin{aligned} (a_1a_2 - a_0a_3) =& \ D_1^*(D_2^*)^2\left(\frac{r_1r_2^2}{K_1K_2^2} - \frac{r_2}{K_2}c_{12}c_{21}\right) + (D_1^*)^2D_2^*\left(\frac{r_1^2r_2}{K_1^2K_2} - \frac{r_1}{K_1}c_{12}c_{21}\right) + \\ & D_1^*(D_3^*)^2\left(\frac{r_1r_3^2}{K_1K_3^2} - \frac{r_3}{K_3}c_{13}c_{31}\right) + (D_1^*)^2D_3^*\left(\frac{r_1^2r_3}{K_1^2K_3} - \frac{r_1}{K_1}c_{13}c_{31}\right) + \\ & D_2^*(D_3^*)^2\left(\frac{r_2r_3^2}{K_2K_3^2} - \frac{r_3}{K_3}c_{23}c_{32}\right) + D_1^*D_2^*D_3^*\left(\frac{2r_1r_2r_3}{K_1K_2K_3} - c_{12}c_{31}c_{23}\right) \\ & + D_1^*D_2^*D_3^*\left(-c_{21}c_{13}c_{32}\right) > 0. \end{aligned} \tag{10}$$

And therefore by Routh Hurwitz stability criterion the point $P(D_1^*, D_2^*, D_3^*)$ is locally asymptotically stable as we have $\chi > 0$ and $(D_j^*)_{j=1,2,3} > 0$. □

4 Bioeconomic Model of the Three Populations of Fish

The main purpose of this section is to define and study a bioeconomic equilibrium model for two fishermen who catch three fish populations.

The fishing amount can be studied from different aspects, in this context we introduce the fishing by its correlations with the effort, the catchability and population growth rate by the amount:

$$H_{ij} = q_jE_{ij}D_j \ , \tag{11}$$

where $(q_j)_{j=1,2,3}$ are the catchability coefficients of species j; and $(E_j)_{j=1,2,3}$ are the fishing efforts to exploit species j.

This bioeconomic model includes three parts in correlation: a biological part, an exploitation part, and an economic part.

4.1 Biological Part

The aim of the biological part is to connect the catchability to the biomass stock.

We give expression of biomass as a function of fishing effort.

$$\begin{cases} \dot{D}_1 = r_1 D_1 \left(1 - \frac{D_1}{K_1}\right) - c_{12} D_1 D_2 - c_{13} D_1 D_3 - q_1 E_1 D_1 \\ \dot{D}_2 = r_2 D_2 \left(1 - \frac{D_2}{K_2}\right) - c_{21} D_2 D_1 - c_{23} D_2 D_3 - q_2 E_2 D_2 \\ \dot{D}_3 = r_3 D_3 \left(1 - \frac{D_3}{K_3}\right) - c_{31} D_1 D_3 - c_{32} D_2 D_3 - q_3 E_3 D_3 \end{cases} . \tag{12}$$

Remark 2. *In this paper we assume that the catchability coefficient q is a constant.*

The evolution model of fish population becomes:

$$\begin{cases} r_1 \left(1 - \frac{D_1}{K_1}\right) = c_{12} D_2 + c_{13} D_3 + q_1 E_1 \\ r_2 \left(1 - \frac{D_2}{K_2}\right) = c_{21} D_1 + c_{23} D_3 + q_2 E_2 \\ r_3 \left(1 - \frac{D_3}{K_3}\right) = c_{31} D_1 + c_{32} D_2 + q_3 E_3 \end{cases} . \tag{13}$$

The solutions of the system (12) are given by:

$$\begin{cases} D_1 = n_{11} E_1 + n_{12} E_2 + n_{13} E_3 + D_1^* \\ D_2 = n_{21} E_1 + n_{22} E_2 + n_{23} E_3 + D_2^* \\ D_3 = n_{31} E_1 + n_{32} E_2 + n_{33} E_3 + D_3^*, \end{cases} \tag{14}$$

where

$$\begin{aligned} n_{11} &= \frac{-K_1(K_2 K_3 q_1 c_{23} c_{32} - q_1 r_2 r_3)}{\chi} \\ n_{21} &= \frac{K_2(K_1 K_3 q_1 c_{31} c_{23} - K_1 q_1 r_3 c_{21})}{\chi} \\ n_{31} &= \frac{K_3(K_1 K_2 q_1 c_{21} c_{32} - K_1 q_1 r_2 c_{31})}{\chi} \\ n_{12} &= \frac{-K_1(K_2 q_2 r_3 c_{12} - K_2 K_3 q_2 c_{13} c_{32})}{\chi} \\ n_{22} &= \frac{K_2(q_2 r_1 r_3 - K_1 K_3 q_2 c_{13} c_{31})}{\chi} \\ n_{32} &= \frac{K_3(K_1 K_2 q_2 c_{12} c_{31} - K_2 q_2 r_1 c_{32})}{\chi} \\ n_{13} &= \frac{-K_1(K_3 q_3 r_2 c_{13} - K_2 K_3 q_3 c_{12} c_{23})}{\chi} \\ n_{23} &= \frac{K_2(K_1 K_3 q_3 c_{21} c_{13} - K_3 q_3 r_1 c_{23})}{\chi} \\ n_{33} &= \frac{K_3(q_3 r_1 r_2 - K_1 K_2 q_3 c_{12} c_{21})}{\chi}. \end{aligned}$$

We define the matrix D, where $D = -NE + D^*$ and $N = (-n_{ij})_{1 \leq i,j \leq 3}$ and $E = (E_1, E_2, E_3)^T$.

As supposed $r_j r_k > c_{ij} c_{ji} K_i K_j$ for all $j, k = 1, 2, 3$, that implies that $n_{ii} < 0$ for all $i = 1, 2, 3$.

4.2 Exploitation Part

The purpose of the exploitation part is to connect the catch to the fishing effort.

1. The fishing effort is defined as the amount of fishing activity and power.
2. The effort depends on several variables: number of hours spent fishing; number of days spent fishing; technology; number of operations; ship; fishing gear etc.
3. The effort is a unidimensional variable which includes a combination of all these factors.
4. The factors cited above are considered internal elements of our system that are measurable and optimizable in the term of the fishing strategy, there are also external factors that can affect the fishing effort and that are not considered in the marginal measured elements of the fishing strategy.

4.2.1 The Net Economic Revenue

We represent the profit of each fisherman by the following function:

$$\Pi_i(E) = (TR)_i - (TC)_i \ , \tag{15}$$

where $\Pi_i(E)$ is the the profit for each fisherman, $(TR)_i$ is the total revenue and $(TC)_i$ is the total cost. While, the Total Revenue depends lineary on the catch $TR = price \times catches$. The total catches of species j by all fishermen

$$H_j = \sum H_{ij}. \tag{16}$$

The total fishing effort dedicated to species j by all fishermen

$$E_j = \sum E_{ij}. \tag{17}$$

We develop these notations as follow:

$$\begin{aligned}(TR)_i &= p_1H_{i1} + p_2H_{i2} + p_3H_{i3}\\ &= p_1q_1E_{i1}D_1 + p_2q_2E_{i2}D_2 + p_3q_3E_{i3}D_3\\ &= p_1q_1E_{i1}\left(n_{11}E_1 + n_{12}E_2 + n_{13}E_3 + D_1^*\right)\\ &+p_2q_2E_{i2}\left(n_{21}E_1 + n_{22}E_2 + n_{23}E_3 + D_2^*\right)\\ &+p_3q_3E_{i3}\left(n_{31}E_1 + n_{32}E_2 + n_{33}E_3 + D_3^*\right)\\ &= p_1q_1E_{i1}\left(n_{11}\sum_{i=1}^{n}E_{i1} + n_{12}\sum_{i=1}^{n}E_{i2} + n_{13}\sum_{i=1}^{n}E_{i3} + D_1^*\right)\\ &+p_2q_2E_{i2}\left(n_{21}\sum_{i=1}^{n}E_{i1} + n_{22}\sum_{i=1}^{n}E_{i2} + n_{23}\sum_{i=1}^{n}E_{i3} + D_2^*\right)\\ &+p_3q_3E_{i3}\left(n_{31}\sum_{i=1}^{n}E_{i1} + n_{32}\sum_{i=1}^{n}E_{i2} + n_{33}\sum_{i=1}^{n}E_{i3} + D_3^*\right).\end{aligned}$$

Then

$$(TR)_i = \left\langle E^i, -pqNE^i\right\rangle + \left\langle E^i, pqD^* - \sum_{j=1.j\neq 1}^{n} pqNE^j\right\rangle, \tag{18}$$

where $(p_j)_{j=1,2,3}$ is the price per unit biomass of the species j. In this work, we take p_1, p_2 and p_3 as constants.

4.3 Economic Part

The aim of this part is to connect the fishing effort to the profit.

4.3.1 The Total Effort Cost

In consederation of many standard fisheries models, (e.g., the model of (Clark, 1975) and (Gordon, 1954)), we assume that

$$(TC)_i = < c^i, E^i >, \tag{19}$$

where $(TC)_i$ is the total effort cost of the fisherman i, and c^i is the constant cost per unit of harvesting and E^i is the total effort of the fisherman i.

4.3.2 Expression of The Profit

The revenue of each fisherman is represented by the following function

$$\begin{aligned}\Pi_i(E) &= (TR)_i - (TC)_i\\ &= \left\langle E^i, -pqNE^i\right\rangle + \left\langle E^i, pqD^* - \sum_{j=1.j\neq 1}^{n} pqNE^j\right\rangle.\end{aligned} \tag{20}$$

As the biological model is meaningful only if the biomass of all the marine species are strictly positive.

Then $D = -NE + D^* \geq D_0 > 0$.

For each fisherman i we have

$$NE^i \leq -\sum_{j=1.j\neq 1}^{n} NE^j + D^*. \tag{21}$$

4.3.3 Nash Equilibrium

A Nash equilibrium solution exists when the involved fishermen does not change the strategy of fishing; while each fisherman tries to maximize his profit and achieve a fishing effort.

This problem can be translated into an optimization problem:

The first fisherman must solve problem (P_1)

$$\max\ \Pi_1(E) = \left\langle E^1, -pqNE^1 + pqD^* - c - pqNE^2 \right\rangle, \tag{22}$$

As

$$\begin{array}{l} NE^1 \leq -NE^2 + D^*, \\ E^1 \geq 0\ , \\ E^2 \text{ is given }. \end{array}$$

And the second fisherman must solve problem (P_2):

$$\max\ \Pi_2(E) = \left\langle E^2, -pqNE^2 + pqD^* - c - pqNE^1 \right\rangle, \tag{23}$$

As

$$\begin{array}{l} NE^2 \leq -NE^1 + D^*, \\ E^2 \geq 0\ \ , \\ E^1 \text{ is given }. \end{array}$$

The point (E^1, E^2) is a generalized Nash equilibrium, where E^1 is a solution of problem (P_1) for E^2 given, and E^2 is a solution of problem (P_2) for E^1 given.

The solution of the Generalized Nash Equilibrium Problem. By applying Karush-Kuhn-Tucker to problem (P_1) we have $u_1 \in \mathbb{R}^3_+, v_1 \in \mathbb{R}^3_+$ and $\lambda^1 \in \mathbb{R}^3_+$ such that

$$\begin{cases} 2pqNE^1 + c - pqD^* + pqNE^2 - u^1 + N^T\lambda^1 = 0 \\ NE^1 + v^1 = -NE2 + D^* \\ < u1, E1 > \ =< \lambda^1, v1 > \ = 0. \end{cases}$$

In the same way, the conditions of Karush-Kuhn-Tucker applied to problem (P_2), we have $u_2 \in \mathbb{R}^3_+, v_2 \in \mathbb{R}^3_+$ and $\lambda^2 \in \mathbb{R}^3_+$ such that

$$\begin{cases} 2pqNE^2 + c - pqD^* + pqNE^1 - u^1 + N^T\lambda^2 = 0 \\ NE^2 + v^2 = -NE^2 + D^* \\ < u^2, E^2 > \ =< \lambda^2, v^2 > \ = 0. \end{cases} \tag{24}$$

From (23) and (24) we have

$$\begin{cases} u^1 = 2pqNE^1 + c - pqD^* + pqNE^2 + N^T\lambda^1 \\ u^2 = 2pqNE^2 + c - pqD^* + pqNE^1 + N^T\lambda^2 \\ v^1 = -NE^1 - NE^2 + D^* \\ v^2 = -NE^1 - NE^2 + D^* \\ < u^i, E^i > \ =< \lambda^i, v^i > \ = 0 \text{ for all } i = 1,2,3 \quad (*1) \\ E^i, u^i, \lambda^i, vi \geqslant 0 \text{ for all } i = 1,2,3 \qquad (*2) \end{cases} \tag{25}$$

From $(*1)$ and $(*2)$ we have $v^1 = v^2$. And as $D_j > 0$ for all $j = 1, 2, 3$; therefore $v^1 = v^2 > 0$. We also have the scalar product of $(\lambda^i)_{i=1,2,3} = 0$ and $(v^i)_{i=1,2,3} = 0$. We denote by $v = v^1 = v^2$, so we have:

$$\begin{cases} u^1 = 2pqNE^1 + pqNE^2 + c - pqD^* \\ u^2 = 2pqNE^2 + pqNE^1 + c - pqD^* \\ v = -NE^1 - NE^2 + D^* \\ < u^i, E^i > \ = 0 \text{ for all } i = 1, 2, 3 \\ E^i, u^i, vi \geqslant 0 \quad \text{for all } i = 1, 2, 3. \end{cases} \tag{26}$$

So

$$\begin{pmatrix} u^1 \\ u^2 \\ v \end{pmatrix} = \begin{bmatrix} 2pqN & pqN & N^T \\ pqN & 2pqN & 0 \\ -N & -N & 0 \end{bmatrix} \begin{pmatrix} E^1 \\ E^2 \\ 0 \end{pmatrix} + \begin{pmatrix} c - pqD^* \\ c - pqD^* \\ D^* \end{pmatrix}. \tag{27}$$

Linear Complementarity Problem. Considering (27) we have a linear complementarity problem equivalent to the Nash equilibriumn problem $LCP(M, b)$ meaning that $z, w \in \mathbb{R}^6$ in order

$$LCP(M, b) \begin{cases} w = Mz + b \geqslant 0 \\ z, w > 0 \\ z^T w = 0. \end{cases} \tag{28}$$

We verify that the $LCP(M, b)$ has a unique solution by the next Theorem.

Theorem 2. *$LCP(M, b)$ has a unique solution for every b if and only if M is a $P-matrix$.*

Proof. Suppose when the complementary pivot algorithm is applied on the LCP(q, M) it ends in ray termination.

As M is a copositive plus matrix and the system has a feasible solution, the LCP has a solution, this implies that there exists a $z^h \geq 0$, $w^h \geq 0$, $z_0^h \geq 0$ satisfying $w^h = Mz^h + e_n z_0^h$; $w_i^h z_i^h = 0$ for all i.

So $z_i^h \left(M_i.z^h\right) + z_i^h z_0^h = 0$. This implies that $z_i^h \left(M_i.z^h\right) = -z_i^h z_h^0 \leqq 0$ for all i.

So M reverses the sign of $z^h \geq 0$, which is a contradiction to the proprety of the nonreversal sign. So, when the complementary pivot method is applied on the LCP(q, M) associated with a P-matrix, it cannot end in ray termination, it has to terminate with a solution of the LCP. This also proves that every P-matrix is a Q-matrix.

Now we will prove that if M is a P-matrix, for any $q \in \mathbb{R}^n$, the LCP(q, M) has exactly one solution, by induction on n, the order of the problem.

Suppose $n = 1$. $M = (m_{11})$ is a P-matrix, if $m_{11} > 0$. In this case $q = (q_1)$.

If $q_1 \geq 0, (w = (w_1) = (q_1) ; z = (z_1) = (0))$ is the only solution to the LCP(q, M).

If $q_1 < 0, (w = (w_1) = (0); z = (z_1) = (-q_1/m_{11}))$ is the only solution to the LCP(q, M).

Hence the theorem is true for $n = 1$.

Induction Hypothesis:

Suppose any LCP of order $(n - 1)$ or less, associated with a P-matrix, has a unique solution for each of its right hand side constant vectors.

Now we will prove that under the induction hypothesis, the LCP(q, M) where M is a P-matrix of order n, has a unique solution for any $q \in \mathbb{R}^n$.

We have shown above that it has at least one solution, say $(\tilde{w}; \tilde{z})$. For each $j = 1$ to n let $u_j = z_j$, if $\tilde{z}_j > 0$; or w_j otherwise; and let v_j be the complement of u_j.

Then $u = (u_1, \ldots, u_n)$ is a complementary feasible basic vector of variables associated with the Basic feasible solution $(\bar{w}; \bar{z})$.

Obtain the canonical next table with respect to the complementary feasible basic vector u, and suppose it is

$u_1, \ldots, u_n$	$v_1, \ldots, v_n$	q
I	$-\widetilde{M}$	$\tilde{q}$

$\tilde{q} \geqq 0$ by our assumptions here.

The table above can itself be viewed as the LCP$(\tilde{q}, \widetilde{M})$, one solution of this LCP is $(u = \tilde{u} = \tilde{q}; v = \tilde{v} = 0)$.

$\widetilde{M}$ is a Principal pivot transform of M, as LCP$(\tilde{q}, \widetilde{M})$ has a unique solution, $\widetilde{M}$ is also a P-matrix.

So all the principal submatrices of $\widetilde{M}$ are also P-matrices.

So the principal subproblem of the LCP $(\tilde{q}, \widetilde{M})$ in the variables $(u_1, \ldots, u_{i-1}, u_{i+1}, \ldots, u_n)$; $(v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n)$ is an LCP of order $(n-1)$ associated with a P-matrix, and by the induction hypothesis this principal subproblem has a unique solution.

One solution of this principal subproblem is $(\tilde{u}_1, \ldots, \tilde{u}_{i-1}, \tilde{u}_{i+1}, \ldots, \tilde{u}_n; \tilde{v}_1, \ldots, \tilde{v}_{i-1}, \tilde{v}_{i+1}, \ldots, \tilde{v}_n) = (\tilde{q}_1, \ldots, \tilde{q}_{i-1}, \tilde{q}_{i+1}, \ldots, \tilde{q}_n; 0, \ldots, 0, 0, \ldots, 0)$.

If the LCP$(\tilde{q}, \widetilde{M})$, has an alternate solution $(\hat{u}; \hat{v}) \neq (\tilde{u}; \tilde{v})$ in which $\hat{v}_i = 0$, its principal subproblem in the variables $(u_1, \ldots, u_{i-1}, u_{i+1}, \ldots, u_n)$; $(v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n)$ will have an alternate solution $(\hat{u}_1, \ldots, \hat{u}_{i-1}, \hat{u}_{i+1}, \ldots, \hat{u}_n; \hat{v}_1, \ldots, \hat{v}_{i-1}, \hat{v}_{i+1}, \ldots, \hat{v}_n)$, a contradiction.

So, if the LCP$(\tilde{q}, \widetilde{M})$ has an alternate solution $(\hat{u}; \hat{v}) \neq (\tilde{u}; \tilde{v})$, then $\hat{v}_i$ must be strictly positive in it, and by complementarity $\hat{u}_i$ must be zero. Since this holds for each $i = 1$ to $n, \hat{v} > 0$, $\hat{u} = 0$.

So $\hat{u} - \widetilde{M}\hat{v} = \tilde{q}, \hat{u} = 0, \hat{v} > 0$. Since $\tilde{q} \geqq 0$, this implies that $\widetilde{M}\hat{v} = -\tilde{q} \leqq 0$, $\hat{v} > 0$, a contradiction since $\widetilde{M}$ is a P-matrix.

Hence under the induction hypothesis the LCP$(\tilde{q}, \widetilde{M})$ has a unique solution, which implies that the equivalent LCP(q, M) has a unique solution also.

Since this holds for any $q \in \mathbb{R}^n$, under the induction hypothesis, the LCP (q, M) of order n has a unique solution for each $q \in \mathbb{R}^n$ when M is a P-matrix.

Hence, by induction the theorem is true.

$\square$

Remark 3. *If M is a $P-$ matrix then the Nash equilibrium problem has a unique solution.*

Theorem 3. *The matrix*

$$M = \begin{bmatrix} 2pqN & pqN & N^T \\ pqN & 2pqN & 0 \\ -N & -N & 0 \end{bmatrix}. \tag{29}$$

is a $P-$ matrix.

Proof. As we have $n_{ii} < 0$ for all $i = 1, 2, 3$ and $\chi > 0$, we denote by $(M_i)_{i=1,\ldots,9}$ the submatrix of M. Then we obtain

$$\begin{aligned}
det(M_1) &= -2p_1q_1n_{11} > 0, \\
det(M_2) &= 4p_1q_1p_2q_2q_1K_1r_3q_2K_2\chi > 0, \\
det(M_3) &= 8p_1q_1p_2q_2p_3q_3q_3K_3q_1K_1q_2K_2\chi^2 > 0, \\
det(M_4) &= -12n_{11}p_1^2q_1^2p_2q_2p_3q_3q_3K_3q_1K_1q_2K_2\chi^2 > 0, \\
det(M_5) &= 18p_1^2q^21p_2^2q_2^2p_3q_3q_1K_1r_3q_2K_2q_3K_3q_1K_1q_2K_2\chi^3 > 0, \\
det(M_6) &= 27p_1^2q_1^2p_2^2q_2^2p_3^2q_3^2(q_3K_3q_1K_1q_2K_2\chi^2)^2 > 0, \\
det(M_7) &= -9p_1q_1p_2^2q_2^2p_3^2q_3^2n_{11}(q_3K_3q_1K_1q_2K_2\chi^2)^2 > 0, \\
det(M_8) &= 3p_1q_1p_2q_2p_3^2q_3^2q_1K_1r_3q_2K_2\chi(q_3K_3q_1K_1q_2K_2\chi^2)^2 > 0, \\
det(M_9) &= p_1q_1p_2q_2p_3q_3(q_3K_3q_1K_1q_2K^2\chi^2)^3 > 0.
\end{aligned}$$

As $det(M_i)_{i=1,....,9} > 0$ then the matrix M is $P-matrix$. So the linear complementarity problem $LCP(M,b)$ admits one and only one solution representing the Nash equilibrium problem given by

$$\begin{cases} E^1 = \frac{1}{3}N^{-1}(D^* - \frac{c}{pq}) \\ E^2 = \frac{1}{3}N^{-1}(D^* - \frac{c}{pq}) \end{cases}, \tag{30}$$

where

$$N^{-1} = \begin{bmatrix} \frac{r_1}{K_1 q_1} & \frac{c_{12}}{q_1} & \frac{c_{13}}{q_1} \\ \frac{c_{21}}{q_2} & \frac{r_2}{K_2 q_2} & \frac{c_{23}}{q_2} \\ \frac{c_{31}}{q_3} & \frac{c_{32}}{q_3} & \frac{r_3}{K3q3} \end{bmatrix}.$$

$\square$

We conclude the fishing effort that maximizes the profit of the fishermen for catching the three species given by (31)

$$\begin{aligned}
E_{11} =& \ \frac{1}{3}\left[\frac{r_1}{K_1 q_1}[\left(D_1^* - \frac{c_1}{p_1 q_1}\right) + \frac{c_{12}}{q_1}\left(D_2^* - \frac{c_1}{p_2 q_2}\right) + \frac{c_{13}}{q_1}\left(D_3^* - \frac{c_1}{p_3 q_3}\right)\right] \\
E_{12} =& \ \frac{1}{3}\left[\frac{r_2}{K_2 q_2}\left(D_2^* - \frac{c_1}{p_2 q_2}\right) + \frac{c_{21}}{q_2}\left(D_1^* - \frac{c_1}{p_1 q_1}\right) + \frac{c_{23}}{q_2}\left(D_3^* - \frac{c_1}{p_3 q_3}\right)\right] \\
E_{13} =& \ \frac{1}{3}\left[\frac{r_3}{K_3 q_3}\left(D_3^* - \frac{c_1}{p_3 q_3}\right) + \frac{c_{31}}{q_3}\left(D_1^* - \frac{c_1}{p_1 q_1}\right) + \frac{c_{32}}{q_3}\left(D_2^* - \frac{c_1}{p_2 q_2}\right)\right] \\
E_{21} =& \ \frac{1}{3}\left[\frac{r_1}{K_1 q_1}\left(D_1^* - \frac{c_2}{p_1 q_1}\right) + \frac{c_{12}}{q_1}\left(D_2^* - \frac{c_2}{p_2 q_2}\right) + \frac{c_{13}}{q_1}\left(D_3^* - \frac{c_2}{p_3 q_3}\right)\right] \\
E_{22} =& \ \frac{1}{3}\left[\frac{r_2}{K_2 q_2}\left(D_2^* - \frac{c_2}{p_2 q_2}\right) + \frac{c_{21}}{q_2}\left(D_1^* - \frac{c_2}{p_1 q_1}\right) + \frac{c_{23}}{q_2}\left(D_3^* - \frac{c_2}{p_3 q_3}\right)\right] \\
E_{23} =& \ \frac{1}{3}\left[\frac{r_3}{K_3 q_3}\left(D_3^* - \frac{c_2}{p_3 q_3}\right) + \frac{c_{31}}{q_3}\left(D_1^* - \frac{c_2}{p_1 q_1}\right) + \frac{c_{32}}{q_3}\left(D_2^* - \frac{c_2}{p_2 q_2}\right)\right],
\end{aligned} \tag{31}$$

where E_{ij} is the fishing effort for fisherman $i = 1,2$ catching species $j = 1,2,3$.

5 The Profit Estimation Considering Temperature Factor

The aim of this section is to estimate the profit of each fisherman taking in consideration the temperature factor

5.1 Open Dynamical system

The problem of determining the fishing effort that maximizes the profit of each fisherman is solved by a Nash equilibrium problem.

By definition a Nash equilibrium exists when there is no unilateral profitable deviation from any of the fishermen involved. In other words, no fisherman would take a different action as long as every other fishermen's strategy remains the same.

This system (two fishermen exploiting three species) is not considered as a closed one, as the two of the fishermen will not change their strategies, yet there are external factors that can interfere in their profit.

5.1.1 External Factors

The habitat characteristics of fish species distribution require physical factors: temperature, water depth, current, waves, etc., and chemical factors: oxygen levels, dissolved minerals, and other substances in their environment.

These characteristics are also conditions of biomass existence and evolution, in other words, if these factors changes, the species react to these changes in order to secure their existence, by immigrating for example.

In this paper, we will estimate the profit of two fishermen exploiting three species and taking into consideration the temperature factor, as the temperature factor is the first important physical factor that interferes in the catchability and spawning of the three species, and also the activity of the fisherman effort.

"As temperature rises, fish are able to digest food quicker, have more energy, and feed more often. So fish become more active and generally are easier to catch. There is a limit though, as temperatures rise the amount of dissolved oxygen in the water decreases".

To study the temperature estimation, we recall the definition of a Discrete-time Markov chain.

Definition 1. *A discrete-time Markov chain is a sequence of random variables $X_0, X_1, ...,$with the Markov property, known as a stochastic process, in which the value of the next variable depends only on the value of the current variable*

$$Pr(X_{n+1} = x|X_1 = x_1, X_2 = x_2....Xn = x_n) \ = Pr(X_{n+1} = x|X_n = x_n).$$

The probability of X_{n+1} only depends on the probability of X_n.

Definition 2. *A Markov chain is called homogeneous if and only if the transition probabilities are independent of the time, such that :*

$$Pr(X_{n+1} = x|X_n = x_n) \ = Pr(X_1 = j|X_0 = i) \text{ where } j + i = 1.$$

5.2 Study Conditions

1. In the region that we are going to study the temperature factor by the Markov chain is: Morocco Casablanca, Home Anfa Station lat: 33.5893, lon: -7.6624, alt: 59 m.

2. Time range considered is between 12h and 06h.

3. We observe that there are three dominating temperature states in the time range considered: Warm $[24° - 29°]$, Comfortable $[13° - 24°C]$, Cold $[7° - 13°C]$.

In the next, we aim to study the temperature estimation for the year 2023, based on the temperature of the years 2020 and 2021. We divide the year periods as trimestrial periods where we observe that for every trimester there are two dominating states as follows:

- Trimester 1: From 1 September to 30 November: Comfortable, Warm.
- Trimester 2: From 1 December to 28 February: Comfortable, Cold.
- Trimester 3: From 1 March to 31 May: Comfortable, Cold.
- Trimester 4: From 1 June to 31 August: Comfortable, Warm.

5.2.1 Markov Graph

The Markov graph is obtained by determining the global states of a period where s_{ij} is the probability of going from the state i to state j, for $i, j = 1, 2$ depending on the period studied, and obtained by calculating the quotient of the number of passages between two states and the global number of days in a perdiod.

We define the 2×2 stochastic matrix S by

$$S = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}, \tag{32}$$

where s_{ij} are the passage probabilities of going from state i to j, $s_{ij} \in [0,1]$ for $i,j = 1,2$ with $s_{11} + s_{12} = 1$ and $s_{21} + s_{22} = 1$.

We define the probability vector by X_k for $k = 1, ...n$:

$$X_k = \begin{bmatrix} (s_1)_k \\ (s_2)_k \end{bmatrix} \tag{33}$$

with $(s_1)_k$ is the probability of state 1 in a day k, and $(s_2)_k$ is the probability of state 2 in a day k with $(s_1)_k + (s_2)_k = 1$.

As we have $X_{k+1} = SX_k$, then

$$\begin{aligned} &X_1 = SX_0 \\ &X_2 = SX_1 = S \times SX_0 = S^2 X_0. \end{aligned}$$

For n iteration, we obtain

$$X_n = S^n X_0. \tag{34}$$

Remark 4. *We are not interested to determine the eigenvalues and eigenvectors in consequence X_∞. This is a local study of different seasons periods of time that will not preserve the same probability for long term range and there are other improbable natural factors that interfere and would not allow the convergence of the tool.*

5.3 Fishermen response to temperature change

In the fishery field the temperature term can define water temperature and water temperature and both likely affect both fish and fishers (Finnis, 2022; Li, 2021), the effect has emerged from water temperature and weather temperature.

The study made by Fiorella et al. (2021) have proved trough an empirical study using observed, longitudinal, and household-level fish catch and fishing behavior data. The study claims that temperature changing affect the human behavior that consequently affects the fishermen profit.

The behavioral pathway analysis on fishing participation, effort, and gear use demonstrated that an increase in air temperature from 28° to 29°C, reduced the probability of fishing by 6%, while a sustained increase from 29° to 30°C reduced the probability of fishing by 8%.

They claim also that the effects are additive.

We conclude that the effect of temperature on the human behavior, is not about the temperature degree itself, but it is about the changing from one state to another state, which means that the probability of fishing rate distribution is not linearly dependant to the temperature, but it occurs when the temperature change is significant.

In this work, we are interested in the number of the changing states of temperature.

5.3.1 Hypothesis

1. We suppose that the temperature change effect is reversible.

2. We predict the temperature state as a range of temperature degrees. It is difficult from this method to decide the prediction of the exact temperature degree, so we suppose that the probability of fishing rate increase or decrease by 14% when we go from a state to another.

5.4 Trimester 1: from 1 September to 30 November

Markov graph

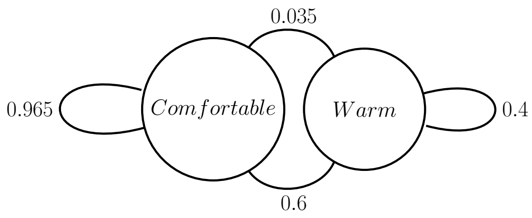

Figure 1: Markov graph of the trimester 1

Stochastic matrix

$$S = \begin{array}{cc} \scriptstyle comfortable & \scriptstyle warm \\ \end{array} \begin{bmatrix} 0.965 & 0.035 \\ 0.6 & 0.4 \end{bmatrix} comfortablewarm \tag{35}$$

Probability vector

As the first day of Septembre 2021 the temperature is comfortable, then the probability vector is

$$X_1 = \begin{bmatrix} (s_1)_1 = 1 \\ (s_1)_2 = 0 \end{bmatrix},$$

where X_1 is the probability vector of the first day of the Trimester. For $n_1 = 1, ..., 91$ we have

$$X_{n_1} = S^{n_1} X_0 \tag{36}$$

Proposition 3. *For the Trimester 1 the Markov Chain achieves equilibrium at* $n = 12$.

Remark 5. *From* $n_1 = 12$ *the distribution is the same for the two states. So* $\Pr(X_{n_{1=13,......91}}) = \frac{1}{2}$ *that means that by the day 12 a significant temperature change occurs between the Comfortable and Warm states that decrease the probability of fishing by 14%.*

5.5 Trimester 2: from 1 December to 28 February

Markov graph

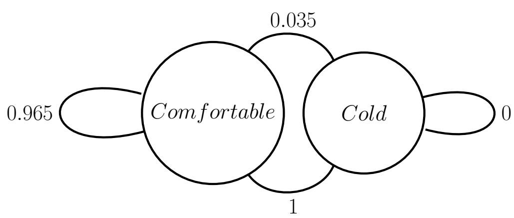

Figure 2: Markov graph of the trimester 2

Stochastic matrix

$$S = \begin{array}{cc} \scriptstyle comfortable & \scriptstyle cold \\ \end{array} \begin{bmatrix} 0.965 & 0.035 \\ 1 & 0 \end{bmatrix} comfortablecold \tag{37}$$

Probability vector

As the first day of December 2021 the temperature is comfortable, then the probability vector is

$$X_1 = \begin{bmatrix} (s_1)_1 = 1 \\ (s_1)_2 = 0 \end{bmatrix},$$

where X_1 is the probability vector of the first day of the Trimester. For $n_2 = 1, ..., 90$ we have

$$X_{n_2} = S^{n_2} X_0 \tag{38}$$

Proposition 4. *For the Trimester 2 the Markov Chain achieves equilibrium at $n = 4$.*

Remark 6. *From $n_2 = 4$ the distribution is the same for the two states. So $\Pr(X_{n_{2=4,......90}}) = \frac{1}{2}$ which means that by day 4 a significant temperature change occur between the Comfortable and Cold states that decrease the probability of fishing by 14%.*

5.6 Trimester 3: from 1 March to 31 May

Markov graph

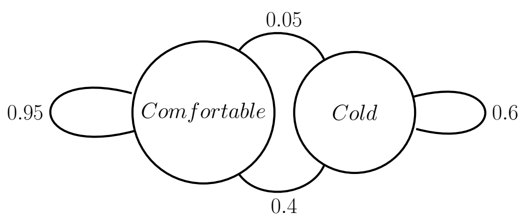

Figure 3: Markov graph of the trimester 3

Stochastic matrix

$$S = \begin{array}{c} \begin{array}{cc} comfortable & cold \end{array} \\ \begin{bmatrix} 0.95 & 0.05 \\ 0.4 & 0.6 \end{bmatrix} \end{array} \; comfortable cold \tag{39}$$

Probability vector

As the first day of March 2022 the temperature is comfortable then the probability vector is

$$X_1 = \begin{bmatrix} (s_1)_1 = 1 \\ (s_1)_2 = 0 \end{bmatrix},$$

where X_1 is the probability vector of the first day of the Trimester. For $n_3 = 1, ..., 92$ we have

$$X_{n_3} = S^{n_3} X_0 \tag{40}$$

Proposition 5. *For the Trimester 3 the Markov Chain achieves equilibrium at $n = 21$.*

Remark 7. *From $n_3 = 21$ the distribution is the same for the the two states then $\Pr(X_{n_{3=21,.......92}}) = \frac{1}{2}$ which means that by day 21 a significant temperature change occur between the Comfortable and Cold states that decrease the probability of fishing by 14%.*

5.7 Trimester 4: from 1 June to 31 August

Markov graph

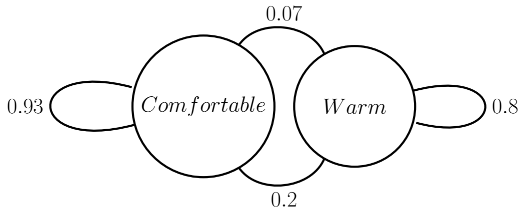

Figure 4: Markov graph of the trimester 4

Stochastic matrix

$$S = \begin{array}{c} \begin{array}{cc} comfortable & warm \end{array} \\ \begin{bmatrix} 0.93 & 0.07 \\ 0.2 & 0.8 \end{bmatrix} \end{array} \; comfortable warm \tag{41}$$

Probability vector
As the first day of June 2022 the temperature is comfortable then the probability vector is

$$X_1 = \begin{bmatrix} (s_1)_1 = 1 \\ (s_1)_2 = 0 \end{bmatrix},$$

where X_1 is the probability vector of the first day of the Trimester. For $n_4 = 1, ..., 91$ we have

$$X_{n_4} = S^{n_4} X_0. \tag{42}$$

Proposition 6. *For Trimester 4 the Markov Chain achieves equilibrium at* $n = 38$.

Remark 8. *From* $n_4 = 38$ *the distribution is the same for the the two states then* $\Pr(X_{n_{4=38,......91}}) = \frac{1}{2}$ *which means that by the day 38 a significant temperature change occur between the Comfortable and Warm states that decrease the probability of fishing by 14%.*

5.8 Spawning period

The period of spawning varies depending on the species, zone, fertility, meteorological conditions,etc.

In order to protect the biodiversity of theses species the fishermen are involved to respect the spawning period, which the temperature factor plays a major role.

6 Simulation

In this section we simulate the model above (estimation of the profit of two fishermen exploiting three fish species in competition considering temperature factor). We consider the parameters of system (3) as revealed in Table 1 in order to assure the existence and stability of the locally asymptotically stable state of the three fish populations, for the economic parameters of system (31) revealed in Table 2, and represent the profit estimation in Table 3.

Table 1: Biological Parameters of the three species.

$Species1$	$Species2$	$Species3$
$r_1 = 0.5$	$r_2 = 0.3$	$r_3 = 0.2$
$K_1 = 1000$	$K_2 = 700$	$K_1 = 600$
$c_{12} = 2.10^{-4}$	$c_{21} = 105$	$c_{31} = 10^{-4}$
$c_{13} = 3.10^{-4}$	$c_{23} = 2.10^{-5}$	$c_{32} = 10^{-4}$

Table 2: Economic Parameters of the mode

$Species1$	$Species2$	$Species3$
$a_1 = 0.1$	$a_2 = 0.2$	$a_3 = 0.3$
$p_1 = 1$	$p_2 = 2$	$p_3 = 3$
$q_1 = 0.1$	$q_2 = 0.02$	$q_3 = 0.004$
$c_1 = 0.1$	$c_1 = 0.1$	$c_1 = 0.1$
$c_2 = 0.2$	$c_2 = 0.2$	$c_2 = 0.2$

Remark 9. *The growth parameters are estimated by Von Bertalanffy for the Eastern Central Atlantic.*

Table 3: The Total profit estimation consedereing the temperature factor.

$Profit$	$Price$	$Tr1\ Profit$	$Tr2\ Profit$	$Tr3\ Profit$	$Tr4\ Profit$	$Total\ Profit$
Π_1 Π_2	$p_1 = 1$ $p_2 = 2$ $p_3 = 3$	70.1 66.9	69.3 66.2	70.8 67.5	69.9 66.7	280.1 267.3
Π_1 Π_2	$p_1 = 11$ $p_2 = 17$ $p_3 = 23$	736.3 732.1	728.2 724	743.3 739	734 729.8	2941.8 2924.9
Π_1 Π_2	$p_1 = 16$ $p_2 = 27$ $p_3 = 48$	1123 1119.8	1110.6 1107.4	1133.7 1130.4	1119.5 1116.3	4486.8 4473.9
Π_1 Π_2	$p_1 = 31$ $p_2 = 47$ $p_3 = 78$	2109.9 2106.4	2086.6 2083.2	2129.9 2126.4	2103.4 2099.9	8429.8 8415.9

Here $Tr1$, $Tr2$, $Tr3$, $Tr4$ are the Trimesters from 1 to 4, and Π_1, Π_2 are the first and second fishermen profit.

These results are different from the results found by (Agmour et al., 2017). So we can assume the impact of the temperature factor on the profit, and else if we take in consideration other significant factors, the results will change, for that we aim to study other factors impact on the fishermen profit.

In other hand, we observe that the Markov chain in some Trimesters reaches quickly the equilibrium which means that the transition probability will remain Invariant, which is a limitation for the tool, and we aim to study the convergence of the temperature estimation made by us with the real temperature for the next year.

Also, as shown in the 4th Trimester the Markov equilibrium didn't occur until the day 38, what make us question if the Trimesters are more detailed and divided will that impact the estimation quality, for that we need to study the convergence of the current Model, to a Model more divided in term of time range, that we suggest to be the perspective of our next work.

7 Disscussion

This work is a case study of profit estimation considering the temperature factor, applied on two fishermen catching three different species in competition, by the Markov chain tool, the question involved through communications about this model is about the choice of estimating the temperature by the Markov chain while we can have free Data on Internet of the Estimation of the temperature, the answer we present is that even those Data on Internet are made by a Mathematic tool, that can be other mathematical probability methods that Markov chain is one of them, and also we aim to study other factors with the same tool so we need we suppose to use the same estimation tool on all potential factors.

8 Conclusion

In this paper, we have developed a Markovian model estimating the temperature factor changes for the year 2023 by taking in consideration the seasons and temperature distribution of it, in the way to estimate the fishing profit of two fishermen catching three species, the temperature factor is a critic element that interfere in the species activities and spawning, and also for the fishermen activities and effort dedicated.

9 Data Availability

The datasets analysed during the current study are available publicly in the websites; [www.onp.ma/] and [www.windguru.cz].

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