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THE PRODUCTION OF CP-ODD HIGGS BOSONS IN POLARIZED ELECTRON-POSITRON COLLISIONS

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Abstract. Within the framework of the Minimal Supersymmetric Standard Model (MSSM), taking into account arbitrary polarization states of the electron-positron pair, analytical expressions of differential effective cross sections of the processes of the production of three CP-odd Higgs bosons and a Z-boson, and two CP-odd Higgs bosons in electron-positron collisions are obtained: $e^-e^+ \to AAA$, $e^-e^+ \to ZAA$. All possible Feyman diagrams with vertices $Z\Phi A$, ZAA and $ZZ\Phi$, ΦAA, $ZZAA$, $Z\Phi A$, ΦZA are taken into account, where $\Phi-$ CP-even Higgs boson – h or H. Left-right (A_{LR}) and transverse (A_φ) spin asymmetries are determined, characteristic features of the behavior of asymmetries and differential effective cross-sections of processes depending on the departure angles and particle energies are investigated. It is shown that the left-right asymmetry A_{LR} depends only on the Weinberg parameter $x_W = \sin^2\theta_W$, and the transverse spin asymmetry A_φ is a function of the departure angles and the particle energy. The possibility of measuring the three-boson interaction constant $\lambda_{\Phi AA}$ and the g_{ZZAA} constant of the interaction of two Z-bosons and two A-bosons is discussed.

Keywords: Minimal Supersymmetric Standard Model, Higgs boson, electron-positron pair, left-right spin asymmetry, transverse spin asymmetry.

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1 Introduction

As is known, the Higgs mechanism (Higgs, 1964a, 1964b; Englert & Brout, 1964) is the key point of the electroweak sector of the Standard Model (SM) (Glashow, 1967; Weinberg, 1967; Salam, 1968). Electroweak gauge bosons and fundamental particles of matter acquire mass due to interaction with a scalar field – the Higgs boson H_{SM}. The discovery of the Higgs boson with characteristics corresponding to SM predictions was carried out by ATLAS and CMS collaborations at the Large Hadron Collider (LHC) (ATLAS Collaboration, 2012; CMS Collaboration, 2012) (see also reviews Rubakov (2012); Lanyov (2014); Kazakov (2014)).

One of the main constants of the Higgs boson interaction is the constant of the three Higgs boson interaction $\lambda_{H_{SM}H_{SM}H_{SM}}$. This constant can be measured during the production of two Higgs bosons and a Zboson in electron-positron collisions: $e^-e^+ \to ZH_{SM}H_{SM}$. This process without taking into account the polarization states of the electron-positron pair is considered in (Djouadi, 2005). We have already investigated this process taking into account arbitrary polarization states of an electron-positron pair (Abdullayev & Gojayev, 2022, 2023).

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Along with SM, is also widely discussed in the literature the MSSM (Djouadi, 2003; Djouadi et al., 1999; Kazakov, 2001). Here we introduce two doublets of the scalar field φ_1 and φ_2, the potential energy, which is expressed as (Djouadi et al., 1999) (CP-preserving theory)

$$\begin{aligned} V(\varphi_1,\varphi_2) = & \, m_{11}^2(\varphi_1^+\varphi_1) + m_{22}^2(\varphi_2^+\varphi_2) \\ & - \left[m_{12}^2(\varphi_1^+\varphi_2) + \text{h.c.}\right] \\ & + \frac{1}{2}\lambda_1(\varphi_1^+\varphi_1)^2 + \frac{1}{2}\lambda_2(\varphi_2^+\varphi_2)^2 \\ & + \lambda_3(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) + \lambda_4(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1) \\ & + \left\{\frac{1}{2}\lambda_5(\varphi_1^+\varphi_2)^2 + \left[\lambda_6(\varphi_1^+\varphi_1) + \lambda_7(\varphi_2^+\varphi_2)\right](\varphi_1^+\varphi_2) + \text{h.c.}\right\}. \end{aligned} \tag{1}$$

In the MSSM, the parameters $\lambda_1 - \lambda_7$ are given by the constants g and g' of the electroweak symmetry $SU_L(2) \times U_Y(1)$:

$$\begin{aligned} &\lambda_1 = \lambda_2 = \frac{1}{4}\left(g^2 + g'^2\right), \qquad \lambda_3 = \frac{1}{4}(g^2 - g'^2), \\ &\lambda_4 = -\frac{1}{2}g^2, \qquad \lambda_5 = \lambda_6 = \lambda_7 = 0, \end{aligned} \tag{2}$$

and the mass parameters are equal:

$$\begin{aligned} m_{11}^2 &= \left(M_A^2 + M_Z^2\right)\sin^2\beta - \frac{1}{2}M_Z^2, \\ m_{22}^2 &= \left(M_A^2 + M_Z^2\right)\cos^2\beta - \frac{1}{2}M_Z^2, \\ m_{12}^2 &= \frac{1}{2}M_A^2\sin 2\beta. \end{aligned} \tag{3}$$

Here M_A and M_Z – are the masses of A and Z-bosons, β – is the mixing angle of the scalar fields of the MSSM.

The scalar fields φ_1 and φ_2 are decomposed into real and imaginary parts around the vacuum state

$$\begin{aligned} \varphi_1 &= \frac{1}{\sqrt{2}}\begin{pmatrix} \upsilon_1 + H_1^0 + iP_1^0 \\ H_1^- \end{pmatrix}, \\ \varphi_2 &= \frac{1}{\sqrt{2}}\begin{pmatrix} H_2^+ \\ \upsilon_2 + H_2^0 + iP_2^0 \end{pmatrix}. \end{aligned} \tag{4}$$

Physically, CP-even Higgs bosons H and h are obtained by mixing the fields H_1^0 and H_2^0 (mixing angle α):

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix}. \tag{5}$$

By mixing the fields P_1^0 and P_2^0 ($H_1^\pm$ and $H_2^\pm$), the CP-odd Higgs boson A (charged Higgs bosons $H^\pm$) is obtained (here the mixing angle β):

$$\begin{aligned} \begin{pmatrix} G^0 \\ A \end{pmatrix} &= \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} P_1^0 \\ P_2^0 \end{pmatrix}, \\ \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} &= \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} H_1^\pm \\ H_2^\pm \end{pmatrix}, \end{aligned} \tag{6}$$

where G^0 and $G^\pm$– are neutral and charged Goldstone bosons.

Higgs bosons are characterized by six parameters: M_H, M_h, M_A, $M_{H^\pm}$, α and β. Of these, only two parameters are free: M_A and $tan\beta$. The masses M_H and M_h are defined through the masses M_A and M_Z and by the parameter $tan\beta$:

$$M^2_{H,h} = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}\right]. \qquad (7)$$

The mass of charged Higgs bosons is expressed by the masses M_A and M_W:

$$M^2_{H^\pm} = M_A^2 + M_W^2. \qquad (8)$$

The mixing angle α is given by the expression

$$\tan 2\alpha = \tan 2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2} \quad \left(-\frac{\pi}{2} \le \alpha \le 0\right). \qquad (9)$$

Detection of Higgs bosons $H, h, A, H^\pm$ and determination of their physical characteristics is one of the main tasks of the LHC collider and future electron-positron or muon-antimuon colliders. Currently, the construction of a new generation of electron-positron colliders ILC, CLIC, FCC-ee, CEPS has already been designed (Shiltsev, 2012; Peters, 2017). These colliders in the future will allow us to study the physical properties of Higgs bosons MSSM.

Note that the main processes that can occur in electron-positron collisions are the production of three CP-odd Higgs bosons:

$$e^- + e^+ \to A + A + A \qquad \text{(I)}$$

and the joint production of a vector Z-boson and two CP-odd A-bosons

$$e^- + e^+ \to Z + A + A. \qquad \text{(II)}$$

These processes are considered in (Djouadi et al, 1999), however, the polarization states of the electron-positron pair are not taken into account in this work, the angular and energy distributions of CP-odd Higgs bosons are not investigated. In this paper, we have investigated the processes (I) and (II) taking into account the annihilation of an arbitrarily polarized electron-positron pair. Analytical expressions for the differential effective cross sections of processes (I) and (II) are obtained within the framework of the MSSM. Left-right A_{LR} and transverse A_φ spin asymmetries due to electron-positron pair polarizations are determined. The dependence of asymmetries and differential effective cross sections on the departure angles and particle energies is studied in detail. The possibility of measuring the constant of the three Higgs boson interaction $\lambda_{\Phi AA}$ and the constant of the interaction of two vector Z-bosons and two CP-odd A-bosons g_{ZZAA} is discussed.

2 The production of three CP-odd Higgs bosons

First, let s consider the process of the production of three CP-odd Higgs bosons in electron-positron collisions. It is known that the vertices Zhh, ZhH, ZHH and ZAA are forbidden by CP-invariance in the MSSM. Possible vertices are ZHA and ZhA. Consequently, the process of the production of three CP-odd A-bosons in the collision of an electron-positron pair is described by the Feynman diagrams shown in Fig. 1. According to these diagrams, the electron-positron pair annihilates into a vector Z^*-boson, which turns into a CP-odd A-boson and a CP-even Φ^*-boson ($\Phi^* = h^*$- or H^*-boson), and then the Φ^*-boson decays for two A-bosons.

In the MSSM, the amplitude corresponding to diagram a) of Fig. 1 can be written as follows

$$M_a = g_{Zee} g_{Z\Phi A} g_{\Phi AA} \ell_\mu D_{\mu\nu}(p)[k_{3\nu} - (p - k_3)_\nu] D_\Phi(p - k_3)\varphi^*(k_1)\varphi^*(k_2)\varphi^*(k_3), \qquad (10)$$

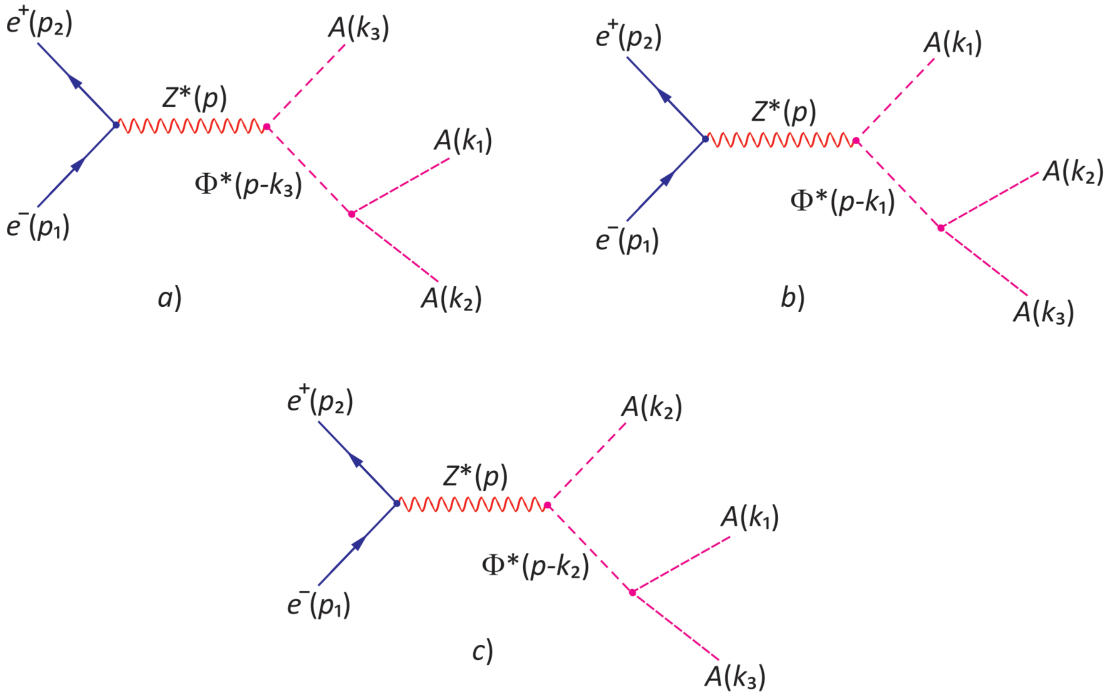

Figure 1: Feynman diagrams of the reaction $e^-e^+ \to AAA$

where $p = p_1 + p_2$, p_1, p_2 and k_1, k_2, k_3- 4-momentum of the electron, positron and CP-odd A-bosons; $g_{Zee} = (\sqrt{2}G_F)^{1/2}M_Z-$ is the interaction constant of an electron with a vector Z-boson; $g_{Z\Phi A}-$ is the interaction constant of a Z-boson with Φ- and A-bosons; $\ell_\mu-$ is the weak neutral current of an electron-positron pair:

$$\ell_\mu = \bar{\upsilon}(p_2, s_2)\gamma_\mu[g_L(1+\gamma_5) + g_R(1-\gamma_5)]u(p_1, s_1); \tag{11}$$

s_1 and s_2 – are the 4-polarization vectors of the electron and positron

$$g_L = -\frac{1}{2} + x_W, \quad g_R = x_W \tag{12}$$

the left and right constants of the interaction of an electron with a Z-boson; $x_W = \sin^2\theta_W$ – the Weinberg parameter; $D_{\mu\nu}(p)$ and $D_\Phi(p - k_3)$ – propagators of vector Z- and Φ-bosons

$$D_{\mu\nu}(p) = i\frac{-g_{\mu\nu} + p_\mu p_\nu/M_Z^2}{p^2 - M_Z^2}, \tag{13}$$

$$D_\Phi\left(p - k_3\right) = \frac{i}{(p-k_3)^2 - M_\Phi^2};$$

$M_\Phi-$ is the mass of the Φ-boson; $g_{\Phi AA}-$ is the interaction constant of CP-even Φ and CP-odd A-bosons; $\varphi^*(k_i)(i = 1,\ 2,\ 3)-$is the unit-normalized wave function of the A-boson with a 4-momentum k_i.

At high energies of the electron-positron pair $s >> m_e^2$ (where $s-$ is the square of the total energy e^-e^+-pairs in the center of mass system, m_e- is the mass of the electron), a weak neutral current is preserved:

$$\ell_\mu p_\mu = \ell_\mu(p_1 + p_2)_\mu = 0,$$

as a result, the amplitude (10) is simplified:

$$M_a = -ig_{Zee}g_Z\frac{M_Z^2}{v}\frac{1}{s^2(1-r_Z)}C_3(\ell_\nu k_{3\nu})\varphi^*(k_1)\varphi^*(k_2)\varphi^*(k_3). \tag{14}$$

It is taken into account here that (Djouadi et al, 1999)

$$\begin{aligned} g_{ZhA} &= \frac{g_Z}{2}\cos(\beta-\alpha), \\ g_{ZHA} &= -\frac{g_Z}{2}\sin(\beta-\alpha), \\ g_{hAA} &= -i\frac{M_Z^2}{\upsilon}\lambda_{hAA}, \\ g_{HAA} &= -i\frac{M_Z^2}{\upsilon}\lambda_{HAA} \end{aligned} \tag{15}$$

and the notation is derived

$$\begin{aligned} &C_i = \frac{\lambda_{hAA}\cos(\beta-\alpha)}{y_i+r_A-r_h} - \frac{\lambda_{HAA}\sin(\beta-\alpha)}{y_i+r_A-r_H}, \quad (i=1,2,3) \\ &r_Z = \frac{M_Z^2}{s}, \quad r_A = \frac{M_A^2}{s}, \quad r_\Phi = \frac{M_\Phi^2}{s}\,(\Phi = h, H), \\ &y_i = 1-x_i, \quad x_i = \frac{2E_i}{\sqrt{s}}; \end{aligned} \tag{16}$$

E_i- is the energy of the A-boson with 4-momentum k_i; $\upsilon = (\sqrt{2}G_F)^{-1/2} = 246$ GeV is the vacuum value of the standard Higgs boson field; $g_Z = g/\cos\theta_W$; the three-boson interaction constants λ_{hAA} and λ_{HAA} are functions of the mixing angles of the scalar fields α and β (Djouadi et al., 1999):

$$\lambda_{hAA} = \cos 2\beta\cdot\sin(\beta+\alpha), \quad \lambda_{HAA} = -\cos 2\beta\cdot\cos(\beta+\alpha).$$

Diagrams b) and c) Fig. 1 differ from diagram a) by swapping the Higgs bosons $A(k_3)\to A(k_2)$ and $A(k_3)\to A(k_1)$. The total amplitude of the process $e^-e^+\to AAA$ consists of three amplitudes:

$$\begin{aligned} M = &-ig_{Zee}g_Z\frac{M_Z^2}{\upsilon}\frac{1}{s^2(1-r_Z)}\ell_\nu \\ &\times[C_3k_{3\nu}+C_2k_{2\nu}+C_1k_{1\nu}]\varphi^*(k_1)\varphi^*(k_2)\varphi^*(k_3). \end{aligned} \tag{17}$$

Squaring the amplitude modulus (17):

$$|M|^2 = \frac{8\sqrt{2}G_F^3M_Z^6}{s^3(1-r_Z)^2}\cdot r_Z L_{\mu\nu}\cdot C_{\mu\nu}, \tag{18}$$

where $L_{\mu\nu}-$ is the electron-positron tensor:

$$\begin{aligned} L_{\mu\nu} = &\,2(g_L^2+g_R^2)[p_{1\mu}p_{2\nu}+p_{2\mu}p_{1\nu}-(p_1\cdot p_2)g_{\mu\nu} \\ &-m_e^2(s_{1\mu}s_{2\nu}+s_{2\mu}s_{1\nu}-(s_1\cdot s_2)g_{\mu\nu})] \\ &+2(g_L^2-g_R^2)m_e[p_{1\mu}s_{2\nu}+s_{2\mu}p_{1\nu}-(p_1\cdot s_2)g_{\mu\nu} \\ &-p_{2\mu}s_{1\nu}-s_{1\mu}p_{2\nu}+(p_2\cdot s_1)g_{\mu\nu})] \\ &+4g_Lg_R[-(p_1\cdot p_2)(s_{1\mu}s_{2\nu}+s_{2\mu}s_{1\nu}-(s_1\cdot s_2)g_{\mu\nu}) \\ &-(s_1\cdot s_2)(p_{1\mu}p_{2\nu}+p_{2\mu}p_{1\nu}) \\ &+(p_2\cdot s_1)(p_{1\mu}s_{2\nu}+s_{2\mu}p_{1\nu}-(p_1\cdot s_2)g_{\mu\nu}) \\ &+(p_1\cdot s_2)(s_{1\mu}p_{2\nu}+p_{2\mu}s_{1\nu})], \end{aligned} \tag{19}$$

$C_{\mu\nu}-$ the tensor of CP-odd Higgs bosons

$$\begin{aligned} C_{\mu\nu} = &\,C_3^2k_{3\mu}k_{3\nu}+C_2^2k_{2\mu}k_{2\nu}+C_1^2k_{1\mu}k_{1\nu}+C_3C_2\,(k_{3\mu}k_{2\nu}+k_{2\mu}k_{3\nu}) \\ &+C_3C_1\,(k_{3\mu}k_{1\nu}+k_{1\mu}k_{3\nu})+C_2C_1\,(k_{2\mu}k_{1\nu}+k_{1\mu}k_{2\nu})\,. \end{aligned} \tag{20}$$

Note that in the electron-positron tensor (19) we have left only the symmetric part, since the tensor $C_{\mu\nu}$ is symmetric.

After multiplying the tensors $L_{\mu\nu}$ and $C_{\mu\nu}$, for the square of the amplitude modulus (18), we obtain the expression:

$$
\begin{aligned}
|M|^2 = \frac{16\sqrt{2}G_F^3 M^6 r_z}{s^3(1-r_z)^2} \cdot r_z \Bigg\{ & C_3^2\left[g_L^2 + g_R^2\right] \cdot \\
& \times \Big[2(p_1\cdot k_3)(p_2\cdot k_3) - (p_1\cdot p_2)M_A^2 - m_e^2\big(2(s_1\cdot k_3)(s_2\cdot k_3) - (s_1\cdot s_2)M_A^2\big)\Big] \\
& + (g_L^2 - g_R^2)m_e\Big[2(p_1\cdot k_3)(s_2\cdot k_3) - (p_1\cdot s_2)M_A^2 - 2(p_2\cdot k_3)(s_1\cdot k_3) + (p_2\cdot s_1)M_A^2\Big] \\
& + 2g_L g_R\Big[-(p_1\cdot p_2)\big(2(s_1\cdot k_3)(s_2\cdot k_3) - (s_1\cdot s_2)M_A^2\big) - 2(s_1\cdot s_2)(p_1\cdot k_3)(p_2\cdot k_3) \\
& \quad + (p_2\cdot s_1)\big(2(p_1\cdot k_3)(s_2\cdot k_3) - (p_1\cdot s_2)M_A^2\big) + 2(p_1\cdot s_2)(s_1\cdot k_3)(p_2\cdot k_3)\Big] \\
& + C_2^2[k_3\to k_2] + C_1^2[k_3\to k_1] \\
& + 2C_3C_2\big(g_L^2 + g_R^2\big)\Big[(p_1\cdot k_3)(p_2\cdot k_2) + (p_1\cdot k_1)(p_2\cdot k_3) - (p_1\cdot p_2)(k_2\cdot k_3) \\
& \quad - m_e^2\big((s_1\cdot k_3)(s_2\cdot k_2) + (s_1\cdot k_2)(s_2\cdot k_3) - (s_1\cdot s_2)(k_2\cdot k_3)\big)\Big] \\
& + (g_L^2 - g_R^2)m_e\Big[(p_1\cdot k_3)(s_2\cdot k_2) + (p_1\cdot k_2)(s_2\cdot k_3) - (p_1\cdot s_2)(k_2\cdot k_3) \\
& \quad - (p_2\cdot k_2)(s_1\cdot k_3) + (p_2\cdot s_1)(k_2\cdot k_3)\Big] \\
& + 2g_L g_R\Big[-(p_1\cdot p_2)\big((s_1\cdot k_3)(s_2\cdot k_2) + (s_1\cdot k_2)(s_2\cdot k_3)\big) \\
& \quad - (s_1\cdot s_2)\big((p_1\cdot k_3)(p_2\cdot k_2) + (p_1\cdot k_2)(p_2\cdot k_3)\big) \\
& \quad + (p_2\cdot s_1)\big((p_1\cdot k_3)(s_2\cdot k_2) + (p_1\cdot k_2)(s_2\cdot k_3)\big) \\
& \quad + (p_1\cdot s_2)\big((s_1\cdot k_3)(p_2\cdot k_2) + (s_1\cdot k_2)(p_2\cdot k_3)\big)\Big] \\
& + 2C_3C_1[k_2\to k_1] + 2C_2C_1[k_3\to k_1]\Bigg\}.
\end{aligned}
\tag{21}
$$

The unit spin vectors of the electron and positron in their rest systems $\vec{\xi}_1$ and $\vec{\xi}_2$ are decomposed into longitudinal and transverse components:

$$
\vec{\xi}_1 = \vec{n}\lambda_1 + \vec{\eta}_1, \quad \vec{\xi}_2 = -\vec{n}\lambda_2 + \vec{\eta}_2, \tag{22}
$$

where $\vec{n}$ – is a unit vector directed by the electron momentum, λ_1 and λ_2 – are the helicities of the electron and positron, $\vec{\eta}_1$ and $\vec{\eta}_2$ – are the transverse components of their spin vectors.

First, let s assume that the electron-positron pair is longitudinally polarized. In this case, the differential effective cross section of the reaction $e^-e^+ \to AAA$ will take the form:

$$
\begin{aligned}
\frac{d\sigma(\lambda_1,\lambda_2)}{dx_1 dx_2 d\Omega} = \frac{G_F^3 M_Z^6}{248\sqrt{2}\pi^4} \cdot \frac{r_Z}{s^2(1-r_Z)^2} \Big[& g_L^2(1-\lambda_1)(1+\lambda_2) \\
& + g_R^2(1+\lambda_1)(1-\lambda_2)F_1\Big],
\end{aligned}
\tag{23}
$$

where

$$F_1 = C_3^2\left[x_3^2(1-\upsilon^2\cos^2\theta_3)-4r_A\upsilon^2\sin^2\theta_3\right]+C_2^2\left[x_2^2(1-\upsilon^2\cos^2\theta_2)\right.$$
$$\left.-4r_A\upsilon^2\sin^2\theta_2\right]+C_1^2\left[x_1^2(1-\upsilon^2\cos^2\theta_1)-4r_A\upsilon^2\sin^2\theta_1\right]$$
$$+2C_3C_2\left[x_2x_3-\upsilon^2\sqrt{(x_2^2-4r_A)(x_3^2-4r_A)}\cdot\cos\theta_2\cos\theta_3-2(y_1-r_A)\right]$$
$$+2C_3C_1\left[x_1x_3-\upsilon^2\sqrt{(x_1^2-4r_A)(x_2^2-4r_A)}\cdot\cos\theta_1\cos\theta_3-2(y_2-r_A)\right]$$
$$+2C_2C_1\left[x_1x_2-\upsilon^2\sqrt{(x_1^2-4r_A)(x_2^2-4r_A)}\cdot\cos\theta_1\cos\theta_2-2(y_3-r_A)\right], \quad (24)$$

$\upsilon=\sqrt{1-4m_e^2/s}$ – is the velocity of the electron, $d\Omega-$ solid angle of departure A- boson with 4-momentum k_1; θ_1,θ_2 and θ_3 – the angles between the directions of the momenta of the electron and the Higgs bosons with momenta $\overrightarrow{k}_1$, $\overrightarrow{k}_2$ and $\overrightarrow{k}_3$.

It follows from the formula of the differential effective cross section (23) that the electron and positron must have opposite helicities: $e_L^-e_R^+$ and $e_R^-e_L^+$. This is due to the law of conservation of the total moment in the transition $e^-e^+\to Z^*$. Therefore, the process $e^-e^+\to AAA$ corresponds to two spiral sections

$$d\sigma(e_L^-e_R^+\to AAA)=\frac{G_F^3M_Z^6}{62\sqrt{2}\pi^4}\cdot\frac{r_Zg_L^2}{s(1-r_Z)^2}F_1dx_1dx_2d\Omega,$$

$$d\sigma(e_R^-e_L^+\to AAA)=\frac{G_F^3M_Z^6}{62\sqrt{2}\pi^4}\cdot\frac{r_Zg_R^2}{s(1-r_Z)^2}F_1dx_1dx_2d\Omega.$$

So, the process $e^-e^+\to AAA$ has a left-right spin asymmetry

$$A_{LR}=\frac{d\sigma(e_L^-e_R^+\to AAA)-d\sigma(e_R^-e_L^+\to AAA)}{d\sigma(e_L^-e_R^+\to AAA)+d\sigma(e_R^-e_L^+\to AAA)}=\frac{g_L^2-g_R^2}{g_L^2+g_R^2}=\frac{1-4x_W}{1-4x_W+8x_W^2},$$

depending only on the Weinberg parameter x_W. With the value of this parameter $x_W=0.2315$, the left-right spin asymmetry is equal to: $A_{LR}=0.14$.

Note that in the centre-of-mass e^-e^+-pair for $\overrightarrow{p}_1+\overrightarrow{p}_2=\overrightarrow{k}_1+\overrightarrow{k}_2+\overrightarrow{k}_3=0$, the Higgs bosons are in the same plane with the azimuthal angle φ of departure. In this system, the laws of conservation of energy and momentum in the variables x_1,x_2,x_3 and departure angles $\theta_1,\theta_2,\theta_3$ are written as follows:

$$x_1+x_2+x_3=2,$$
$$\sqrt{x_1^2-4r_A}\cos\theta_1+\sqrt{x_2^2-4r_A}\cos\theta_2+\sqrt{x_3^2-4r_A}\cos\theta_3=0. \quad (25)$$

It follows from these conservation laws that the scaling energy of each of the Higgs bosons varies within

$$\frac{2M_A}{\sqrt{s}}\le x_i\le 1-3r_A \quad (i=1,\ 2,\ 3). \quad (26)$$

Now suppose that the electron-positron pair is transversely polarized. Let s choose a coordinate system so that the momentum of the electron $\overrightarrow{p}_1$ is directed along the Z axis, and its spin vector $\overrightarrow{\eta}_1$ along the X axis (see Fig. 2), then the spin vector of the positron $\overrightarrow{\eta}_2$ will lie in the XOY plane, the angle between the transverse spin vectors $\overrightarrow{\eta}_1$ and $\overrightarrow{\eta}_2$ is denoted by φ_0. In this case, the differential effective cross section of the process $e^-e^+\to AAA$ is expressed by the formula

$$\frac{d\sigma(\overrightarrow{\eta}_1,\overrightarrow{\eta}_2)}{dx_1dx_2d\Omega}=\frac{G_F^3M_Z^6}{248\sqrt{2}\pi^4}\cdot\frac{r_Z}{s(1-r_Z)^2}\left[\left(g_L^2+g_R^2\right)F_1+2g_Lg_R\eta_1\eta_2F_2\right], \quad (27)$$

where the function F_1 is given by the expression (24), and the function F_2 is equal to (the square of the electron velocity is assumed to be $v^2 \approx 1$):

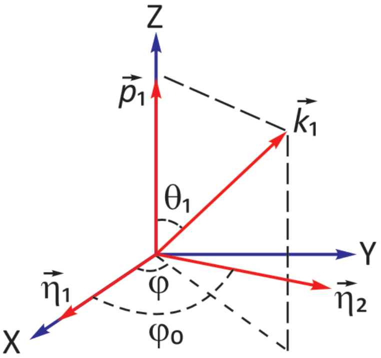

Figure 2: Choosing a coordinate system.

$$\begin{aligned}
F_2 = &-\Big[C_3^2(x_3^2-4r_A)\sin^2\theta_3 + C_2^2(x_2^2-4r_A)\sin^2\theta_2 - C_1^2(x_1^2-4r_A)\sin^2\theta_1\Big] \\
&\times\cos(2\varphi-\varphi_0) + 2C_3C_2\Big[\cos\varphi_0\Big(x_2x_3 - \sqrt{(x_2^2-4r_A)(x_3^2-4r_A)}\cos(\theta_2-\theta_3)\Big) \\
&- 2(y_1-r_A)\sqrt{(x_2^2-4r_A)(x_3^2-4r_A)}\sin\theta_2\sin\theta_3\cos(2\varphi-\varphi_0)\Big] \\
&+ 2C_3C_1\Big[\cos\varphi_0\Big(x_1x_3 - \sqrt{(x_1^2-4r_A)(x_3^2-4r_A)}\cdot(\cos\theta_1\cos\theta_3+\sin\theta_1\sin\theta_3)\Big) \\
&- 2(y_2-r_A)\sqrt{(x_1^2-4r_A)(x_3^2-4r_A)}\sin\theta_1\sin\theta_3\cos(2\varphi-\varphi_0)\Big] \\
&+ 2C_2C_1\Big[\cos\varphi_0\Big(x_1x_2 - \sqrt{(x_1^2-4r_A)(x_2^2-4r_A)}\cdot(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2)\Big) \\
&- 2(y_3-r_A)\sqrt{(x_1^2-4r_A)(x_2^2-4r_A)}\sin\theta_1\sin\theta_2\cos(2\varphi-\varphi_0)\Big].
\end{aligned} \tag{28}$$

It follows from the formula of the differential effective cross section (27) that the process under consideration $e^-e^+ \to AAA$ has a transverse spin asymmetry

$$A_\varphi = \frac{2g_Lg_R}{g_L^2+g_R^2}\cdot\frac{F_2}{F_1}. \tag{29}$$

To estimate the transverse spin asymmetry, we assume that the angle $\varphi_0 = \pi$, the energy of the electron-positron pair $\sqrt{s}$ =500 GeV, the mass of the A-boson M_A = 150 GeV, the parameter $tan\beta = 3$, the Weinberg parameter $x_W = 0.2315$, the scaling energies of the Higgs bosons $x_1 = x_2 = 0.65$ and $x_3 = 0.7$. In addition, we believe that the $\theta_3 = 90°$, and the angles θ_1 and θ_2 associated ratios $\cos\theta_2 = -\cos\theta_1$, $\sin\theta_2 = \sin\theta_1$. In this case, the functions F_1 and F_2 are simplified:

$$\begin{aligned}
F_1 = &C_3^2(x_3^2-4r_A) + (C_2^2+C_1^2)(x_1^2-4r_A)\sin^2\theta_1 + 2C_2C_3\left[x_2x_3 - 2(y_1-r_A)\right] \\
&+ 2C_3C_1\left[x_1x_3 - 2(y_2-r_A)\right] + 2C_2C_1\left[x_1x_2 + (x_1^2-4r_A)\cos^2\theta_1 - 2(y_3-r_A)\right],
\end{aligned} \tag{30}$$

$$\begin{aligned}
F_2 = &\left[C_3^2(x_3^2-4r_A) + (C_2^2+C_1^2)(x_1^2-4r_A)\sin^2\theta_1\right]\cos 2\varphi \\
&+ 2C_2C_3\Big[-x_2x_3 + \sqrt{(x_2^2-4r_A)(x_3^2-4r_A)}\sin\theta_1 - 2(y_1-r_A) \\
&\quad + \sqrt{(x_2^2-4r_A)(x_3^2-4r_A)}\cdot\sin\theta_1\cos 2\varphi\Big] \\
&+ 2C_3C_1\left[-x_1x_3 + (x_1^2-4r_A)(-\cos^2\theta_1 + \sin^2\theta_1 + \sin^2\theta_1\cos 2\varphi) + 2(y_3-r_A)\right].
\end{aligned} \tag{31}$$

Figure 3 shows the angular dependence of the transverse spin asymmetry at the above values of the process parameters $e^-e^+ \to AAA$. As follows from the figure, the transverse spin asymmetry is negative and decreases with an increase in the departure angle θ_1.

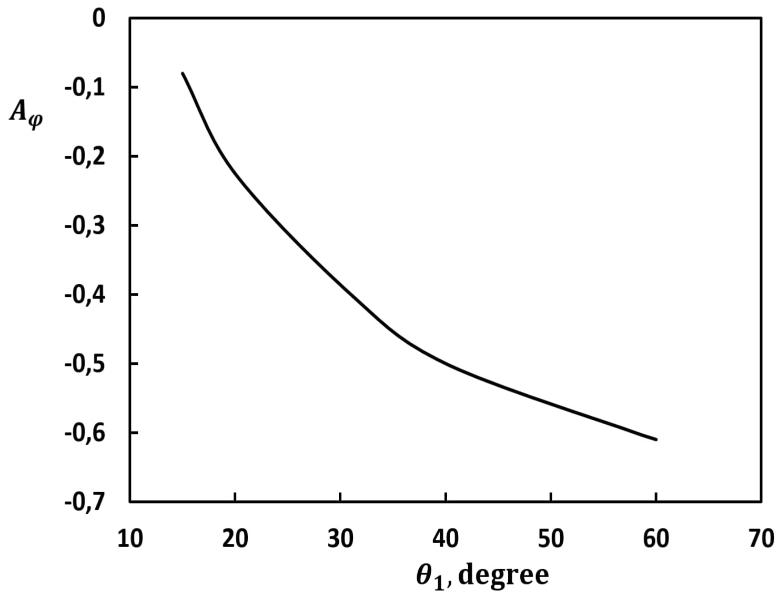

Figure 3: Dependence of the transverse spin asymmetry on the angle θ_1

Averaging over the spin states of the electron-positron pair, for the differential effective cross section of the reaction $e^-e^+ \to AAA$, we obtain the formula

$$\frac{d\sigma}{dx_1 dx_2 d\Omega} = \frac{G_F^3 M_Z^6}{248\sqrt{2}\pi^4} \cdot r_Z \frac{g_L^2 + g_R^2}{s(1-r_Z)^2} F_1, \tag{32}$$

where the function F_1 is given by the formula (24).

Having carried out integrations along the angles of departure of particles, for the energy distribution of Higgs bosons we obtain the formula

$$\begin{aligned} \frac{d\sigma}{dx_1 dx_2} = & \frac{G_F^3 M_Z^6}{96\sqrt{2}\pi^3} \cdot \frac{g_L^2 + g_R^2}{s(1-r_Z)^2} r_Z \\ & \times \left[C_3^2 g_0 + C_2^2 g_1 + C_1^2 g_2 - 2C_3C_2g_3 - 2C_3C_1g_4 + 2C_2C_1g_5\right], \end{aligned} \tag{33}$$

where

$$\begin{aligned} g_0 &= (y_1 + y_2)^2 - 4r_A, \\ g_1 &= y_2(y_2 - 2) - 4r_A + 1, \\ g_2 &= y_1(y_1 - 2) - 4r_A + 1, \\ g_3 &= y_2(y_2 - 1) + y_1(y_1 + 2) - 2r_A, \\ g_4 &= y_1(y_1 - 1) + y_2(y_2 + 1) - 2r_A, \\ g_5 &= y_1 + y_2 + y_1 y_2 + 2r_A - 1. \end{aligned} \tag{34}$$

Figure 4 illustrates the dependence of the differential effective cross section of the reaction $e^-e^+ \to AAA$ on the variable x_1 at $\sqrt{s} = 500$ GeV, $M_A = 150$ GeV, $tan\beta = 3$, $x_W = 0.2315$ and of the scaling energy $x_2 = 0.4$. As can be seen from the figure at a fixed energy $x_2 =$ 0.4, with an increase in the variable x_1, the differential cross-section of the reaction under consideration first increases and reaches a maximum at $x_1 =$ 0.615, and a further increase in the scaling energy leads to a decrease in the effective cross-section.

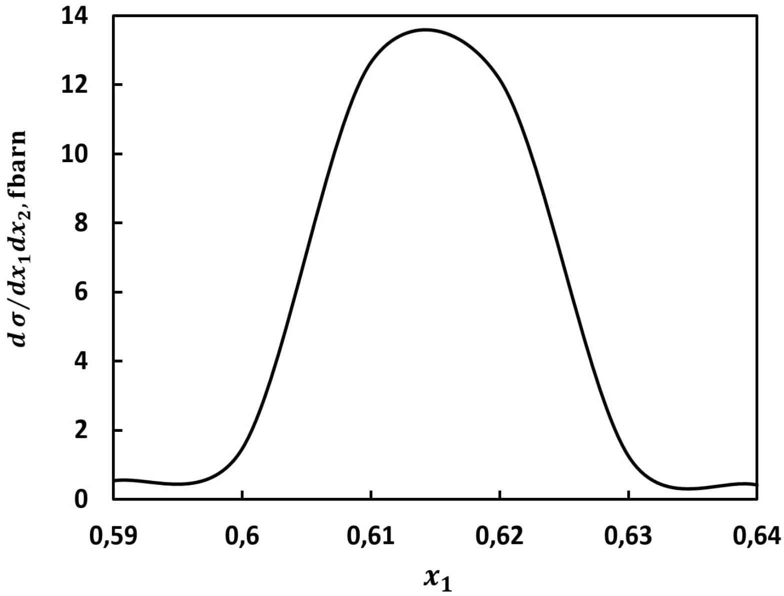

Figure 4: Dependence cross section of the reaction $e^-e^+ \to AAA$ on the variable x_1 at $x_2 = 0.4$.

With another fixed value of the scaling energy $x_2 = 0.45$, the nature of the dependence of the differential effective cross-section of the process $e^-e^+ \to AAA$ on the energy x_1 changes dramatically. Here, a monotonous decrease in the differential cross-section is observed with an increase in the scaling energy x_1 (Fig. 5).

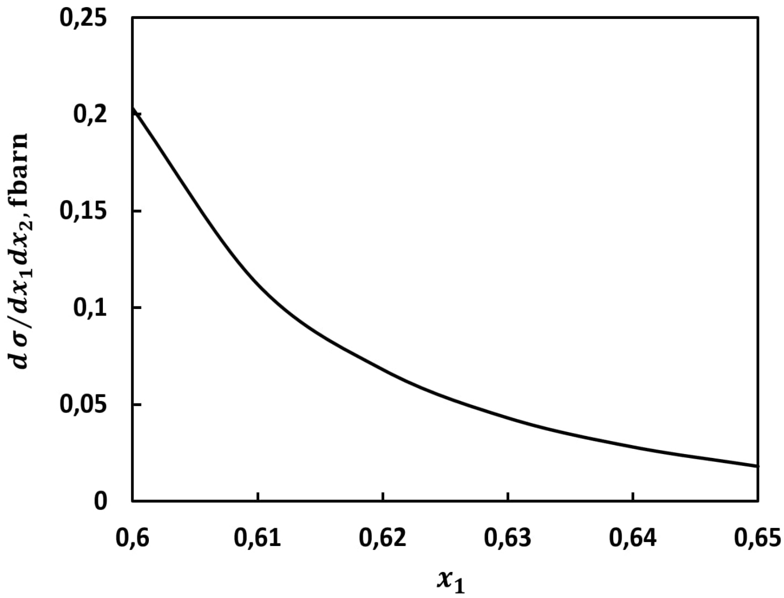

Figure 5: Dependence cross section of the reaction $e^-e^+ \to AAA$ on the variable x_1 at $x_2 = 0.45$.

It should be noted that the experimental study of the reaction of the production of three CP-odd Higgs bosons in electron-positron collisions is of particular interest, since it allows us to accurately measure the constants of the three-boson interactions λ_{hAA} and λ_{HAA}.

3 The production of the Z-boson and two CP-odd Higgs bosons

Now let s consider the process of joint generation of a vector Z-boson and two CP-odd A-bosons in polarized electron-positron collisions $e^-e^+ \to ZAA$. The Feynman diagrams shown in Fig. 6 correspond to this process.

According to diagram a) Fig. 6, the electron-positron pair annihilates into a vector Z^*-boson, which emits a CP-even Φ^*-boson ($\Phi^* = h^*$ or H^*-boson), and the Φ^*-boson turns into two CP-odd A-boson.

In the MSSM, the amplitude corresponding to diagram a) of Fig. 6 can be represented as:

$$M_a = g_{Zee} g_{\Phi ZZ} g_{\Phi AA} \ell_\mu D_{\mu\nu}(p) U_\nu^*(k) D_\Phi(p-k) \varphi^*(k_1)\varphi^*(k_2), \tag{35}$$

where $g_{\Phi ZZ}$ and $g_{\Phi AA}-$ are the interaction constants of the CP-even Φ-boson with Z-bosons and the Φ-boson with CP-odd A-bosons. In the case of $\Phi = h$ and H-bosons, these constants are equal to (Djouadi et al., 1999):

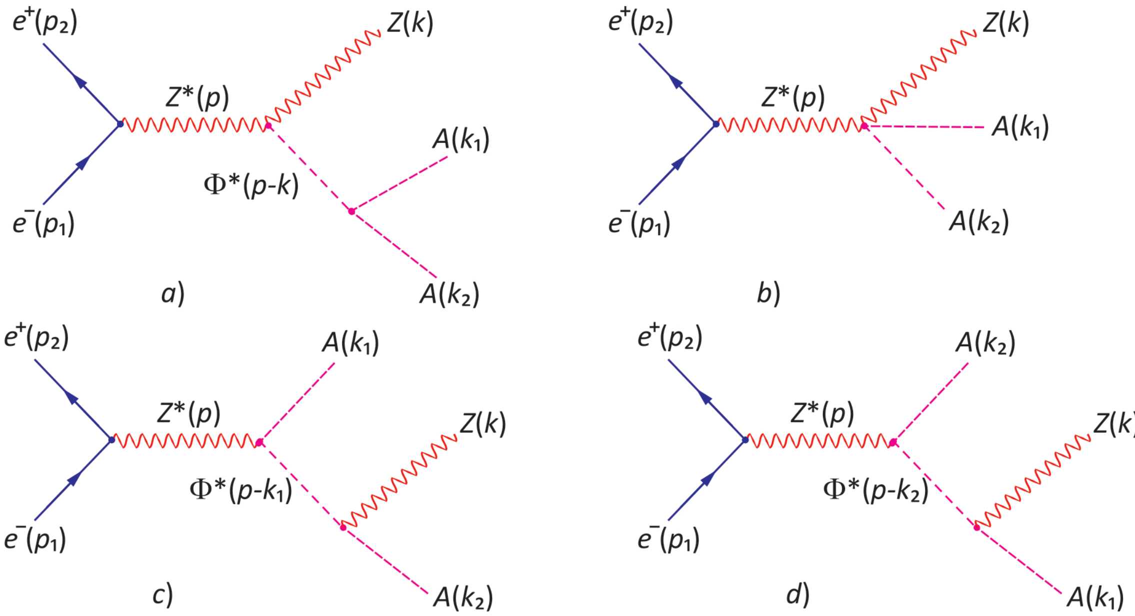

Figure 6: Feynman diagrams of the $e^-e^+ \to ZAA$ process.

$$\begin{aligned} g_{hZZ} &= ig_Z M_Z \sin(\beta-\alpha), \\ g_{hAA} &= -i\frac{M_Z^2}{v}\lambda_{hAA}, \\ g_{HZZ} &= ig_Z M_Z \cos(\beta-\alpha), \\ g_{HAA} &= -i\frac{M_Z^2}{v}\lambda_{HAA}. \end{aligned} \tag{36}$$

Due to the conservation of the electron-positron current $\ell_\mu p_\mu = \ell_\mu(p_{1\mu} + p_{2\mu}) = 0$, the amplitude (35) is simplified

$$\begin{aligned} M_a = g_{Zee} g_Z \frac{M_Z^3}{v} \frac{\ell_\nu U_\nu^*(k)}{s^2(1-r_Z)} \times \\ \times \left[\frac{\lambda_{hAA}\sin(\beta-\alpha)}{y_Z + r_Z - r_h} + \frac{\lambda_{HAA}\cos(\beta-\alpha)}{y_Z + r_Z - r_H} \right] \varphi^*(k_1)\varphi^*(k_2). \end{aligned} \tag{37}$$

Here $y_Z = 1 - x_Z$, $x_Z = 2E_Z/\sqrt{s}$, E_Z- is the energy of the Z-boson and the notation is accepted:

$$r_Z = \frac{M_Z^2}{s}, \quad r_h = \frac{M_h^2}{s}, \quad r_H = \frac{M_H^2}{s}.$$

Now let's write the amplitude corresponding to diagram b) Fig. 6 (according to this diagram, the e^-e^+-pair turns into a vector Z-boson, and that emits two A-bosons at one point and goes into the final state):

$$\begin{aligned} M_b &= g_{Zee} g_{ZZAA} \ell_\mu D_{\mu\nu}(p) U_\nu^*(k) \varphi^*(k_1) \varphi^*(k_2) \\ &= g_{Zee} \frac{g_Z^2}{2} \frac{\ell_\nu U_\nu^*(k)}{s^2(1-r_Z)} \varphi^*(k_1) \varphi^*(k_2), \end{aligned} \tag{38}$$

and we square the sum of the amplitudes M_a and M_b:

$$|M_a + M_b|^2 = \frac{8\sqrt{2} G_F^3 M_Z^6}{s^2(1-r_Z)^2} r_Z^2 a^2 L_{\mu\nu} \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_Z^2} \right), \tag{39}$$

where

$$a = \frac{\lambda_{hAA} \sin(\beta-\alpha)}{y_Z + r_Z - r_h} + \frac{\lambda_{HAA} \cos(\beta-\alpha)}{y_Z + r_Z - r_H} + \frac{1}{r_Z}, \tag{40}$$

$L_{\mu\nu}-$ electron-positron tensor (19).

The product of electron-positron $L_{\mu\nu}$ and Z-boson $(-g_{\mu\nu} + k_\mu k_\nu / M_Z^2)$ tensors is equal to:

$$\begin{aligned} L_{\mu\nu}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_Z^2} \right) &= 2(g_L^2 + g_R^2)\left[(p_1 \cdot p_2) + \frac{2}{M_Z^2}(p_1 \cdot k)(p_2 \cdot k) - m_e^2((s_1 \cdot s_2) + \frac{2}{M_Z^2}(k \cdot s_1)(k \cdot s_2)) \right] \\ &+ 2(g_L^2 - g_R^2) m_e \left[(p_1 \cdot s_2) - (p_2 \cdot s_1) + \frac{2}{M_Z^2}((p_1 \cdot k)(k \cdot s_2) - (p_2 \cdot k)(k \cdot s_1)) \right] \\ &+ 4 g_L g_R \Big[(p_1 \cdot p_2)(s_1 \cdot s_2) - (p_2 \cdot s_1)(p_1 \cdot s_2) + \frac{2}{M_Z^2}(-(p_1 \cdot k)(p_2 \cdot k)(s_1 \cdot s_2)) \\ &- (p_1 \cdot p_2)(k \cdot s_1)(k \cdot s_2) + (p_2 \cdot k)(p_1 \cdot s_2)(k \cdot s_1) + (p_1 \cdot k)(p_2 \cdot s_1)(k \cdot s_2) \Big]. \end{aligned} \tag{41}$$

We proceed to the calculation of diagram c) Fig. 6, which shows that the e^-e^+-pair first turns into a virtual Z^*-boson, and that turns into a CP-even Φ^*-boson and CP-odd $A(k_2)$-boson, then Φ^*-the boson decays into a vector Z- and CP-odd $A(k_1)$-boson.

The amplitude corresponding to this diagram can be written as:

$$M_c = g_{Zee} g_Z^2 \frac{\ell_\nu k_{1\nu}}{s^2(1-r_Z)} \left[\frac{\sin^2(\beta-\alpha)}{y_1 + r_A - r_h} + \frac{\cos^2(\beta-\alpha)}{y_1 + r_A - r_H} \right] k_{2\rho} U_\rho^*(k) \varphi^*(k_1) \varphi^*(k_2), \tag{42}$$

where $y_1 = 1 - x_1$, $x_1 = 2E_1/\sqrt{s}$, E_1- is the energy of the Higgs boson $A(k_1)$.

The square of the amplitude modulus is equal to:

$$\begin{aligned} |M_c|^2 &= g_{Zee}^2 g_Z^2 \frac{1}{s^4(1-r_Z)^2} \left[\frac{\cos^2(\beta-\alpha)}{y_1 + r_A - r_h} + \frac{\sin^2(\beta-\alpha)}{y_1 + r_A - r_H} \right]^2 \\ &\times k_{2\rho} k_{2\sigma} \left(-g_{\rho\sigma} + \frac{k_\rho k_\sigma}{M_Z^2} \right) L_{\mu\nu} k_{1\mu} k_{1\nu} = \frac{8\sqrt{2} G_F^3 M_Z^6}{s^3(1-r_Z)^2 r_Z} \\ &\times \left[(y_1 - r_Z)^2 - 4 r_Z r_A \right] L_{\mu\nu} k_{1\mu} k_{1\nu} \left[\frac{\cos^2(\beta-\alpha)}{y_1 + r_A - r_h} + \frac{\sin^2(\beta-\alpha)}{y_1 + r_A - r_H} \right]^2. \end{aligned} \tag{43}$$

The product of tensors $L_{\mu\nu}k_{1\mu}k_{1\nu}$ is given by the expression

$$\begin{aligned}L_{\mu\nu}k_{1\mu}k_{1\nu} = {} & 2(g_L^2+g_R^2)\left[2(p_1\cdot k_1)(p_2\cdot k_1)-(p_1\cdot p_2)M_A^2\right.\\ & -m_e^2\left(2(k_1\cdot s_1)(k_1\cdot s_2)-(s_1\cdot s_2)M_A^2\right]\\ & +2(g_L^2-g_R^2)m_e\left[2(p_1\cdot k_1)(s_2\cdot k_1)-(p_1\cdot s_2)M_A^2\right.\\ & \left.-2(p_2\cdot k_1)(k_1\cdot s_1)+(p_2\cdot s_1)M_A^2\right]\\ & +4g_Lg_R\Big[-(s_1\cdot s_2)\left(2(p_1\cdot k_1)(p_2\cdot k_1)-(p_1\cdot p_2)M_A^2\right)\\ & -2(p_1\cdot p_2)(k_1\cdot s_1)(k_1\cdot s_2)\\ & +(p_2\cdot s_1)\left(2(p_1\cdot k_1)(k_1\cdot s_2)-(p_1\cdot s_2)M_A^2\right)\\ & +2(p_2\cdot k_1)(p_1\cdot s_2)(k_1\cdot s_1)\Big].\end{aligned} \tag{44}$$

Let us now consider the interference of diagrams (a)+b)) and c) Fig. 6

$$\begin{aligned}(M_a^+ + M_b^+)M_c + M_c^+(M_a+M_b) = {} & \frac{16\sqrt{2}G_F^3M_Z^6}{s^3(1-r_Z)^2}r_Z\left[\frac{\cos^2(\beta-\alpha)}{y_1+r_A-r_h}+\frac{\sin^2(\beta-\alpha)}{y_1+r_A-r_H}\right]a\\ & \times L_{\mu\nu}\left[-k_{1\mu}k_{2\nu}-k_{2\mu}k_{1\nu}+\frac{y_1-r_Z}{2r_Z}(k_\mu k_{1\nu}+k_\nu k_{1\mu})\right].\end{aligned} \tag{45}$$

Find the product of tensors $L_{\mu\nu}(-k_{1\mu}k_{2\nu}-k_{2\mu}k_{1\nu})$ and $L_{\mu\nu}\frac{y_1-r_Z}{2r_Z}(k_\mu k_{1\nu}+k_\nu k_{1\mu})$

$$\begin{aligned}L_{\mu\nu}(-k_{1\mu}k_{2\nu}-k_{2\mu}k_{1\nu}) = {} & -4(g_L^2+g_R^2)\left[(p_1\cdot k_1)(p_2\cdot k_2)+(p_2\cdot k_1)(p_1\cdot k_2)-(p_1\cdot p_2)(k_1\cdot k_2)\right.\\ & \left.-m_e^2\left((k_1\cdot s_1)(k_2\cdot s_2)+(k_1\cdot s_2)(k_2\cdot s_1)-(s_1\cdot s_2)(k_1\cdot k_2)\right)\right]\\ & -4(g_L^2-g_R^2)m_e\left[(p_1\cdot k_1)(k_2\cdot s_2)+(p_1\cdot k_2)(k_1\cdot s_2)-(p_1\cdot s_2)(k_1\cdot k_2)\right.\\ & \left.-(p_2\cdot k_1)(k_2\cdot s_1)-(p_2\cdot k_2)(k_1\cdot s_1)+(p_2\cdot s_1)(k_1\cdot k_2)\right]\\ & -8g_Lg_R\left[-(s_1\cdot s_2)\left((p_1\cdot k_1)(p_2\cdot k_2)+(p_2\cdot k_1)(p_1\cdot k_2)-(p_1\cdot p_2)(k_1\cdot k_2)\right)\right.\\ & -(p_1\cdot p_2)\left((k_1\cdot s_1)(k_2\cdot s_2)+(k_1\cdot s_2)(k_2\cdot s_1)\right)\\ & +(p_2\cdot s_1)\left((p_1\cdot k_1)(k_2\cdot s_2)+(p_1\cdot k_2)(k_1\cdot s_1)-(p_1\cdot s_2)(k_1\cdot k_2)\right)\\ & \left.+(p_1\cdot s_2)\left((p_2\cdot k_1)(k_2\cdot s_1)+(p_1\cdot k_2)(k_1\cdot s_2)\right)\right];\end{aligned} \tag{46}$$

$$\begin{aligned}L_{\mu\nu}\frac{y_1-r_Z}{2r_Z}(k_\mu k_{1\nu}+k_\nu k_{1\mu}) = {} & \frac{2(y_1-r_Z)}{r_Z}\{(g_L^2+g_R^2)[(p_1\cdot k_1)(p_2\cdot k)+(p_2\cdot k_1)(p_1\cdot k)-(p_1\cdot p_2)(k\cdot k_1)]\\ & -m_e^2[(k_1\cdot s_1)(k\cdot s_2)+(k_1\cdot s_2)(k\cdot s_1)-(s_1\cdot s_2)(k\cdot k_1)]\}\\ & +(g_L^2-g_R^2)m_e\{[(p_1\cdot k_1)(k\cdot s_2)+(k_1\cdot s_2)(p_1\cdot k)-(p_1\cdot k_1)(k\cdot s_1)\\ & -(p_2\cdot k_1)(k\cdot s_1)-(p_2\cdot k)(k_1\cdot s_1)+(p_2\cdot s_1)(k\cdot k_1)]\\ & +2g_Lg_R[-(s_1\cdot s_2)[(p_1\cdot k_1)(p_2\cdot k)+(p_2\cdot k_1)(p_1\cdot k)-(p_1\cdot p_2)(k\cdot k_1)]\\ & -(p_1\cdot p_2)[(k_1\cdot s_1)(k\cdot s_2)+(k_1\cdot s_2)(k\cdot s_1)]\\ & +(p_2\cdot s_1)[(p_1\cdot k_1)(k\cdot s_2)+(p_1\cdot k)(k\cdot s_1)(k_1\cdot s_2)-(p_1\cdot s_2)(k\cdot k_1)]\\ & +(p_1\cdot s_2)[(p_2\cdot k_1)(k\cdot s_1)+(p_2\cdot k)(k_1\cdot s_1)]]\}.\end{aligned} \tag{47}$$

Now we calculate the interference of diagrams c) and d), which differ from each other by replacing the places of CP-odd Higgs bosons $A(k_1)$ and $A(k_2)$:

$$
\begin{aligned}
M_c^\dagger M_d + M_d^\dagger M_c = & \frac{8\sqrt{2}G_F^3 M_Z^6}{s^3(1-r_Z)^2 r_Z}\frac{1}{} \left[\frac{\cos^2(\beta-\alpha)}{y_1+r_A-r_h} + \frac{\sin^2(\beta-\alpha)}{y_1+r_A-r_H}\right] \\
& \times \left[\frac{\cos^2(\beta-\alpha)}{y_2+r_A-r_h} + \frac{\sin^2(\beta-\alpha)}{y_2+r_A-r_H}\right] [(y_1-r_Z)(y_2-r_Z) - 2r_Z(y_Z+r_Z-2r_A)] \\
& \times \{(g_L^2+g_R^2)[(p_1\cdot k_1)(p_2\cdot k_2) + (p_2\cdot k_1)(p_1\cdot k_2) - (p_1\cdot p_2)(k_1\cdot k_2)] \\
& - m_e^2[(k_1\cdot s_1)(k_2\cdot s_2) + (k_1\cdot s_2)(k_2\cdot s_1) - (s_1\cdot s_2)(k_1\cdot k_2)]\} \\
& + (g_L^2-g_R^2)m_e\{[(p_1\cdot k_1)(k_2\cdot s_2) + (p_1\cdot k_2)(k_1\cdot s_2) - (p_1\cdot s_2)(k_1\cdot k_2) \\
& - (p_2\cdot k_1)(k_2\cdot s_1) - (p_2\cdot k_2)(k_1\cdot s_1) + (p_2\cdot s_1)(k_1\cdot k_2)] \\
& + 2g_L g_R[-(s_1\cdot s_2)[(p_1\cdot k_1)(p_2\cdot k_2) + (p_1\cdot k_2)(p_2\cdot k_1) - (p_1\cdot p_2)(k_1\cdot k_2)] \\
& - (p_1\cdot p_2)[(k_1\cdot s_1)(k_2\cdot s_2) + (k_1\cdot s_2)(k_2\cdot s_1)] \\
& + (p_2\cdot s_1)[(p_1\cdot k_1)(k_2\cdot s_2) + (p_1\cdot k_2)(k_1\cdot s_2) - (p_1\cdot s_2)(k_1\cdot k_2)] \\
& + (p_1\cdot s_2)[(p_2\cdot k_1)(k_2\cdot s_1) + (p_2\cdot k_2)(k_1\cdot s_1)]]\}.
\end{aligned} \tag{48}
$$

Similarly, it is easy to calculate the square of the amplitude $|M_d|^2$ and the interference of the diagrams (a)+b) and d)):

$$
\begin{aligned}
|M_d|^2 &= |M_c|^2\,(x_1 \leftrightarrow x_2, y_1 \leftrightarrow y_2)\,, \\
(M_a^+ + M_b^+)M_d + M_d^+(M_a+M_b) &= \left[(M_a^+ + M_b^+)M_c + M_c^+(M_a+M_b)\right](x_1 \leftrightarrow x_2, y_1 \leftrightarrow y_2)\,.
\end{aligned} \tag{49}
$$

The unit spin vectors of the electron and positron in their rest systems $\overrightarrow{\xi}_1$ and $\overrightarrow{\xi}_2$ are decomposed into longitudinal and transverse components (see formula (22)). Then the differential effective cross section of the reaction $e^-e^+ \to ZAA$ is expressed by the formula:

$$
\begin{aligned}
\frac{d\sigma(\lambda_1,\lambda_2;\eta_1,\eta_2)}{dx_1 dx_2 d\Omega_Z} = & \frac{\sqrt{2}G_F^3 M_Z^6}{128\pi^4 s}\cdot\frac{1}{(1-r_Z)^2} \\
& \times \left\{\left[g_L^2(1-\lambda_1)(1+\lambda_2) + g_R^2(1+\lambda_1)(1-\lambda_2)\right]\cdot\psi_1\right. \\
& \left.+2g_L g_R \eta_1\eta_2\psi_2\right\},
\end{aligned} \tag{50}
$$

where $d\Omega_Z = \sin\theta_Z d\theta_Z d\varphi-$ solid angle of departure of the Z-boson,

$$
\begin{aligned}
\psi_1 = & \frac{a^2}{2} r_Z f_0 + 4a r_Z\left[\frac{\cos^2(\beta-\alpha)}{y_1+r_A-r_h} + \frac{\sin^2(\beta-\alpha)}{y_1+r_A-r_H}\right] f_1 \\
& + \frac{1}{4r_Z}\left[\frac{\cos^2(\beta-\alpha)}{y_1+r_A-r_h} + \frac{\sin^2(\beta-\alpha)}{y_1+r_A-r_H}\right] \\
& \times\left[\frac{\cos^2(\beta-\alpha)}{y_2+r_A-r_h} + \frac{\sin^2(\beta-\alpha)}{y_2+r_A-r_H}\right] f_2 \\
& + \frac{1}{r_Z}\left[\frac{\cos^2(\beta-\alpha)}{y_1+r_A-r_h} + \frac{\sin^2(\beta-\alpha)}{y_1+r_A-r_H}\right]^2 f_3 \\
& + \{\theta_1 \leftrightarrow \theta_2, x_1 \leftrightarrow x_2, y_1 \leftrightarrow y_2\},
\end{aligned} \tag{51}
$$

$$\begin{aligned}\psi_2 = {} & \frac{a^2}{2} r_Z h_0 + 4ar_Z \left[\frac{\cos^2(\beta-\alpha)}{y_1 + r_A - r_h} + \frac{\sin^2(\beta-\alpha)}{y_1 + r_A - r_H}\right] h_1 \\ & + \frac{1}{4r_Z}\left[\frac{\cos^2(\beta-\alpha)}{y_1 + r_A - r_h} + \frac{\sin^2(\beta-\alpha)}{y_1 + r_A - r_H}\right] \\ & \times \left[\frac{\cos^2(\beta-\alpha)}{y_2 + r_A - r_h} + \frac{\sin^2(\beta-\alpha)}{y_2 + r_A - r_H}\right] h_2 \\ & + \frac{1}{r_Z}\left[\frac{\cos^2(\beta-\alpha)}{y_1 + r_A - r_h} + \frac{\sin^2(\beta-\alpha)}{y_1 + r_A - r_H}\right]^2 h_3 \\ & + \{\theta_1 \leftrightarrow \theta_2, x_1 \leftrightarrow x_2, y_1 \leftrightarrow y_2\},\end{aligned} \tag{52}$$

$$f_0 = x_Z^2(1 - \upsilon^2\cos^2\theta_Z) + 4r_Z(1+\cos^2\theta_Z),$$

$$f_1 = -x_1x_2 + \upsilon^2\sqrt{(x_1^2 - 4r_A)(x_2^2 - 4r_A)} \cdot \cos\theta_1\cos\theta_2$$

$$+2\upsilon^2(y_Z + r_Z - 2r_A) + \frac{y_1 - r_Z}{2r_Z}[x_Z x_1$$

$$-\upsilon^2\sqrt{(x_Z^2 - 4r_Z)(x_1^2 - 4r_A)} \cdot \cos\theta_Z\cos\theta_1 - 2\upsilon^2(y_1 - r_A)],$$

$$f_2 = [(y_1 - r_Z)(y_2 - r_Z) - 2r_Z(y_Z + r_Z - 2r_A)]$$

$$\times[x_1x_2 - 2\upsilon^2(y_Z + r_Z - 2r_A) - \upsilon^2\sqrt{(x_1^2 - 4r_A)(x_2^2 - 4r_A)} \cdot \cos\theta_1\cos\theta_2],$$

$$f_3 = [(y_1 - r_Z)^2 - 4r_Z r_A]$$

$$\times[x_1^2(1 - \upsilon^2\cos^2\theta_1) - 4r_A\upsilon^2\sin^2\theta_1],$$

$$h_0 = -(x_Z^2 - 4r_Z)\sin^2\theta_Z\cos(2\varphi - \varphi_0),$$

$$h_1 = \cos\varphi_0(-x_1x_2 + \upsilon^2\sqrt{(x_1^2 - 4r_A)(x_2^2 - 4r_A)} \cdot \cos\theta_1\cos\theta_2 + 2\upsilon^2(y_Z + r_Z - 2r_A))$$

$$+\upsilon^2\sqrt{(x_1^2 - 4r_A)(x_2^2 - 4r_A)}\sin\theta_1\sin\theta_2(\cos\varphi_0 + \cos(2\varphi - \varphi_0))$$

$$+\frac{y_1 - r_Z}{2r_Z}[\cos\varphi_0(x_Z x_1 - \upsilon^2\sqrt{(x_Z^2 - 4r_Z)(x_1^2 - 4r_A)}\cos\theta_Z\cos\theta_1 - 2\upsilon^2(y_Z + r_Z - 2r_A))$$

$$-\upsilon^2\sqrt{(x_Z^2 - 4r_Z)(x_1^2 - 4r_A)}\sin\theta_1\sin\theta_2(\cos\varphi_0 + \cos(2\varphi - \varphi_0)),$$

$$h_2 = \cos\varphi_0(x_1x_2 - \upsilon^2\sqrt{(x_1^2 - 4r_A)(x_2^2 - 4r_A)} \cdot \cos\theta_1\cos\theta_2 - 2\upsilon^2(y_Z + r_Z - 2r_A))$$

$$-\upsilon^2\sqrt{(x_1^2 - 4r_A)(x_2^2 - 4r_A)}\sin\theta_1\sin\theta_2(\cos\varphi_0 + \cos(2\varphi - \varphi_0)),$$

$$h_3 = -[(y_1 - r_Z)^2 - 4r_Z r_A][(x_1^2 - 4r_A)\sin^2\theta_1 \cos(2\varphi - \varphi_0)],$$

$\theta_Z, \theta_1(\theta_2)-$ is the angle between the directions of the electron and Z^0-boson momentums, the electron and $A(k_1)(A(k_2))$-boson.

First, let's assume that the electron-positron pair is longitudinally polarized. In this case, the differential effective cross section of the process $e^- e^+ \to ZAA$ will take the form:

$$\frac{d\sigma(\lambda_1,\lambda_2)}{dx_Z dx_1 d\Omega_Z} = \frac{\sqrt{2}G_F^3 M_Z^6}{128\pi^4 s}\cdot\frac{1}{(1-r_Z)^2}\times \\ \times\left[g_L^2(1-\lambda_1)(1+\lambda_2) + g_R^2(1+\lambda_1)(1-\lambda_2)\right]\cdot\psi_1, \tag{53}$$

where the function ψ_1 is equal to:

$$\begin{aligned}\psi_1 = {} & \frac{a^2}{2} r_Z (f_0 + f_0') + 4a r_Z \times \\ & \times\left[\left(\frac{\cos^2(\beta-\alpha)}{y_1 + r_A - r_h} + \frac{\sin^2(\beta-\alpha)}{y_1 + r_A - r_H}\right) f_1 + \right. \\ & \left.\left(\frac{\cos^2(\beta-\alpha)}{y_2 + r_A - r_h} + \frac{\sin^2(\beta-\alpha)}{y_2 + r_A - r_H}\right) f_1'\right] + \\ & \frac{1}{4r_Z}\left[\frac{\cos^2(\beta-\alpha)}{y_1 + r_A - r_h} + \frac{\sin^2(\beta-\alpha)}{y_1 + r_A - r_H}\right] \\ & \times\left[\frac{\cos^2(\beta-\alpha)}{y_2 + r_A - r_h} + \frac{\sin^2(\beta-\alpha)}{y_2 + r_A - r_H}\right](f_2 + f_2') + \\ & \frac{1}{r_Z}\left[\frac{\cos^2(\beta-\alpha)}{y_1 + r_A - r_h} + \frac{\sin^2(\beta-\alpha)}{y_1 + r_A - r_H}\right]^2 f_3 \\ & + \frac{1}{r_Z}\left[\frac{\cos^2(\beta-\alpha)}{y_2 + r_A - r_h} + \frac{\sin^2(\beta-\alpha)}{y_2 + r_A - r_H}\right]^2 f_3'.\end{aligned} \tag{54}$$

Here (the electron velocity is assumed to be $v^2 \approx 1$)

$$\begin{aligned} f_0 &= f_0' = x_Z^2 \sin^2\theta_Z + 4r_Z(1+\cos^2\theta_Z), \\ f_1 &= -x_1 x_2 + \sqrt{(x_1^2 - 4r_A)(x_2^2 - 4r_A)}\cdot\cos\theta_1\cos\theta_2 + 2(y_Z + r_Z - 2r_A) + \\ &\quad + \frac{y_1 - r_Z}{2r_Z}\left[x_Z x_1 - \sqrt{(x_Z^2 - 4r_Z)(x_1^2 - 4r_A)}\cdot\cos\theta_Z\cos\theta_1 - 2(y_2 - r_A)\right], \\ f_1' &= -x_1 x_2 + \sqrt{(x_1^2 - 4r_A)(x_2^2 - 4r_A)}\cdot\cos\theta_1\cos\theta_2 + 2(y_Z + r_Z - 2r_A) + \\ &\quad + \frac{y_2 - r_Z}{2r_Z}\left[x_Z x_2 - \sqrt{(x_Z^2 - 4r_Z)(x_2^2 - 4r_A)}\cdot\cos\theta_Z\cos\theta_2 - 2(y_1 - r_A)\right], \\ f_2 &= f_2' = [(y_1 - r_Z)(y_2 - r_Z) - 2r_Z(y_Z + r_Z - 2r_A)]\times \\ &\quad\times\left[x_1 x_2 - \sqrt{(x_1^2 - 4r_A)(x_2^2 - 4r_A)}\cdot\cos\theta_1\cos\theta_2 - 2(y_Z + r_Z - 2r_A)\right], \\ f_3 &= \left[(y_1 - r_Z)^2 - 4r_Z r_A\right](x_1^2 - 4r_A)\sin^2\theta_1, \\ f_3' &= \left[(y_2 - r_Z)^2 - 4r_Z r_A\right](x_2^2 - 4r_A)\sin^2\theta_2. \end{aligned} \tag{55}$$

The left-right spin asymmetry A_{LR}, due to the longitudinal polarization of the electron, depends only on the Weinberg parameter x_W

$$A_{LR} = \frac{1/4 - x_W}{1/4 - x_W + 2x_W^2} \tag{56}$$

and with the value of this parameter, $x_W = 0.2315$ is $A_{LR} = 14\%$.

Now suppose that the electron-positron pair is transversely polarized. In this case, the differential effective cross section of the reaction $e^-e^+ \to ZAA$ is expressed by the formula (angle φ_0 is taken by π and $\upsilon^2 \approx 1$):

$$\frac{d\sigma(\eta_1,\eta_2)}{dx_1 dx_2 d\Omega_Z} = \frac{\sqrt{2}G_F^3 M_Z^6}{128\pi^4 s} \cdot \frac{1}{(1-r_Z)^2}\left[(g_L^2+g_R^2)\cdot\psi_1 + 2g_L g_R \eta_1\eta_2\psi_2\right], \tag{57}$$

where function ψ_1 is expressed by formula (54), and function ψ_2 is equal to

$$\begin{aligned}\psi_2 = &\frac{a^2}{2}(h_0+h_0^{'}) + 4ar_Z\times\\ &\times\left[\left(\frac{\cos^2(\beta-\alpha)}{y_1+r_A-r_h}+\frac{\sin^2(\beta-\alpha)}{y_1+r_A-r_H}\right)h_1 + \left(\frac{\cos^2(\beta-\alpha)}{y_2+r_A-r_h}+\frac{\sin^2(\beta-\alpha)}{y_2+r_A-r_H}\right)h_1^{'}\right]+\\ &\times\frac{1}{r_Z}\left[\left(\frac{\cos^2(\beta-\alpha)}{y_1+r_A-r_h}+\frac{\sin^2(\beta-\alpha)}{y_1+r_A-r_H}\right)\cdot\left(\frac{\cos^2(\beta-\alpha)}{y_2+r_A-r_h}+\frac{\sin^2(\beta-\alpha)}{y_2+r_A-r_H}\right)^2(h_2+h_2^{'})\right]+\\ &+\frac{1}{r_Z}\left[\frac{\cos^2(\beta-\alpha)}{y_1+r_A-r_h}+\frac{\sin^2(\beta-\alpha)}{y_1+r_A-r_H}\right]^2 h_3+\\ &+\frac{1}{r_Z}\left[\frac{\cos^2(\beta-\alpha)}{y_2+r_A-r_h}+\frac{\sin^2(\beta-\alpha)}{y_2+r_A-r_H}\right]^2 h_3^{'}.\end{aligned} \tag{58}$$

Here the functions are introduced:

$$\begin{aligned}h_0 = h_0^{'} &= (x_Z^2-4r_Z)\sin^2\theta_Z\cos 2\varphi,\\ h_1 &= x_1x_2 - \sqrt{(x_1^2-4r_A)(x_2^2-4r_A)}\cdot\cos\theta_1\cos\theta_2 - 2(y_Z+r_Z-2r_A)-\\ &-\sqrt{(x_1^2-4r_A)(x_2^2-4r_A)}\sin\theta_1\sin\theta_2(1+\cos 2\varphi)+\\ &+\frac{y_1-r_Z}{2r_Z}\left[-x_Zx_1+\sqrt{(x_Z^2-4r_Z)(x_1^2-4r_A)}\cos\theta_Z\cos\theta_1+\right.\\ &\left.2(y_Z+r_Z-2r_A)+\sqrt{(x_Z^2-4r_Z)(x_1^2-4r_A)}\sin\theta_Z\sin\theta_1(1+\cos 2\varphi)\right],\end{aligned} \tag{59}$$

$$\begin{aligned}h_1^{'} &= h_1(\theta_1\leftrightarrow\theta_2, x_1\leftrightarrow x_2, y_1\leftrightarrow y_2),\\ h_2 = h_2^{'} &= -x_1x_2+\sqrt{(x_1^2-4r_A)(x_2^2-4r_A)}\cdot\cos\theta_1\cos\theta_2+\\ &+2(y_Z+r_Z-2r_A)+\sqrt{(x_1^2-4r_A)(x_2^2-4r_A)}\sin\theta_1\sin\theta_2(1+\cos 2\varphi),\\ h_3 &= \left[(y_1-r_Z)^2-4r_Zr_A\right](x_1^2-4r_A)\sin^2\theta_1\cos 2\varphi,\\ h_3^{'} &= \left[(y_2-r_Z)^2-4r_Zr_A\right](x_2^2-4r_A)\sin^2\theta_2\cos 2\varphi.\end{aligned}$$

Based on the differential effective cross-section formula (57), we determine the transverse spin asymmetry A_φ due to the transverse polarizations of the electron-positron pair:

$$A_\varphi = \frac{2g_Lg_R}{g_L^2+g_R^2}\cdot\frac{\psi_2}{\psi_1}. \tag{60}$$

The transverse spin asymmetry A_φ, in contrast to the left-right spin asymmetry A_{LR}, depends on the angles of departure of particles and on their energies. In the system of the center of mass of an electron-positron pair, the laws of conservation of energy and momentum in the process $e^-e^+ \to ZAA$ in the scaling variables x_Z, x_1, x_2 and the angles of departure of particles θ_Z, θ_1 and θ_2 are written as follows:

$$x_Z + x_1 + x_2 = 2,$$

$$\sqrt{x_Z^2 - 4r_Z}\cos\theta_Z + \sqrt{x_1^2 - 4r_A}\cos\theta_1 + \sqrt{x_2^2 - 4r_A}\cos\theta_2 = 0.$$

It follows from these conservation laws that the scaling energy of the Z-boson varies in the region

$$\frac{2M_Z}{\sqrt{s}} \le x_Z \le 1 + r_Z - 4r_A.$$

With an energy of e^-e^+-pairs $\sqrt{s} = 500$ GeV, with a mass of $M_A = 150$ GeV and $M_Z = 91.1875$, we have

$$0.36475 \le x_Z \le 0.67326.$$

Figure 7 shows the dependence of the transverse spin asymmetry A_φ on the departure angle θ_Z at scaling energies $x_2 = 0.6$ and $x_Z = 0.435$. As can be seen from the figure, at the beginning of the angular spectrum, the transverse spin asymmetry is positive and decreases with an increase in the angle θ_Z and reaches a minimum value at an angle of 90°, and with a further increase in the angle θ_Z, the asymmetry begins to grow.

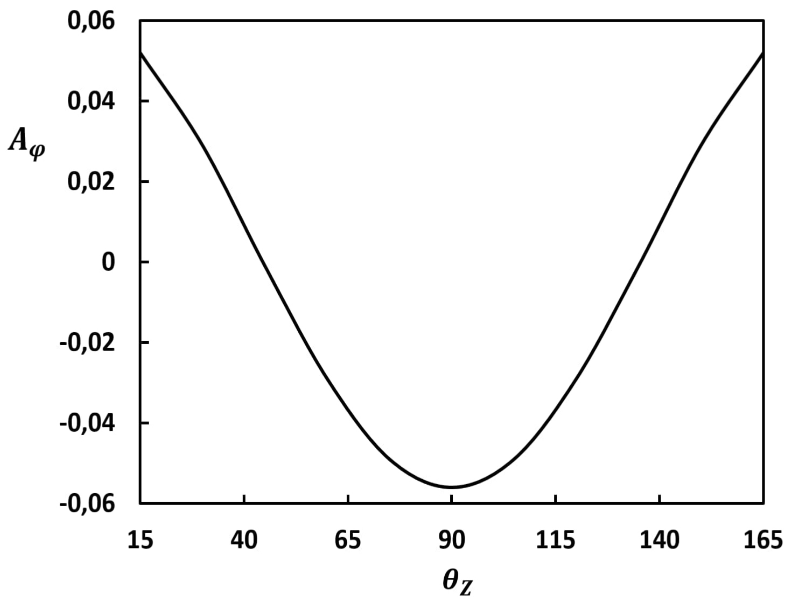

Figure 7: Angular dependence of the transverse spin asymmetry A_φ in the reaction $e^-e^+ \to ZAA$

Figure 8 illustrates the dependence of the transverse spin asymmetry A_φ on the scaling energy x_Z at $x_2 = 0.6$, $\theta_Z = 30°$ and $\varphi = 0°$. The figure shows that at $x_Z =$ 0.4, the transverse spin asymmetry is -0.53 and with an increase in the scaling energy, this value increases and reaches a maximum value of $A_\varphi = 0.4$ at $x_Z = 0.425$. A further increase in the scaling energy of x_Z leads to a decline in the transverse spin asymmetry. At $x_Z = 0.65$, the transverse spin asymmetry is equal to $A_\varphi = 0.018$.

Averaging over the spin states of e^-e^+-pairs, for the differential effective cross section of the reaction $e^-e^+ \to ZAA$, we obtain the formula

$$\frac{d\sigma}{dx_1 dx_2 d\Omega_Z} = \frac{\sqrt{2}G_F^3 M_Z^6}{128\pi^4 s} \cdot \frac{g_L^2 + g_R^2}{(1 - r_Z)^2}\psi_1, \tag{61}$$

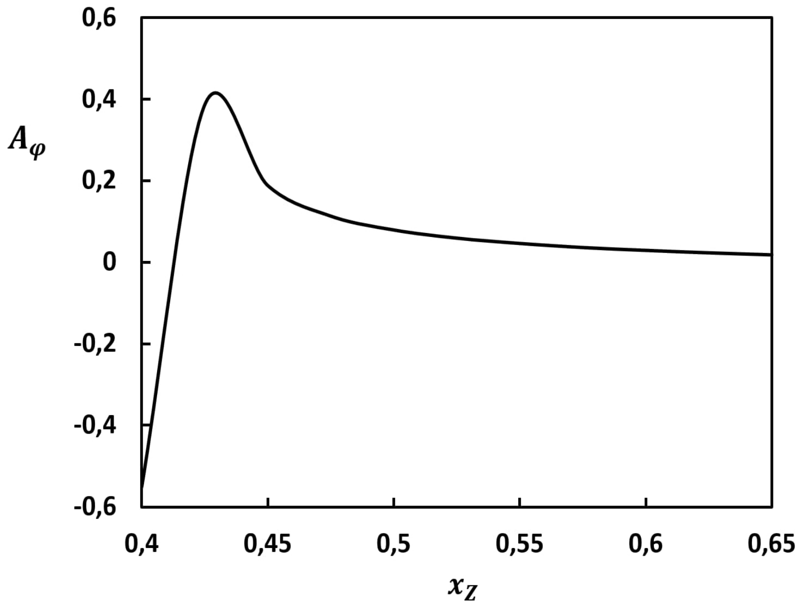

Figure 8: The energy dependence of the transverse spin asymmetry A_φ in the reaction $e^-e^+ \to ZAA$

where the function ψ_1 is given by the expression (54).

Figure 9 shows the dependence of the differential effective cross section of the reaction $e^-e^+ \to ZAA$ on the departure angle θ_Z at the energy of the electron-positron pair $\sqrt{s} = 500$ GeV and scaling energies $x_Z = x_2 = 0.6$. As can be seen from the figure, with an increase in the departure angle θ_Z, the differential effective cross section decreases and reaches a minimum near the angle is 90°, and then begins to grow with increasing angle θ_Z.

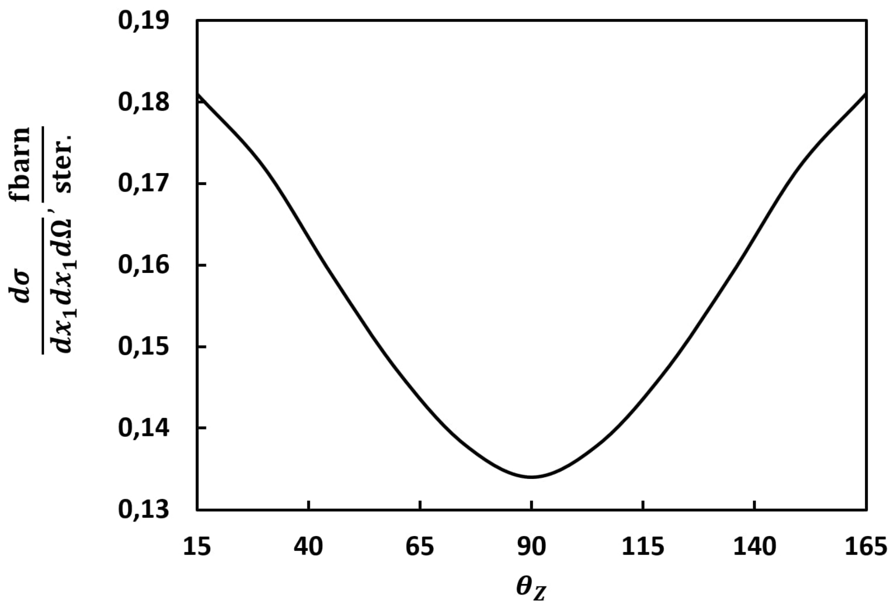

Figure 9: Angular dependence of the differential effective cross section of the reaction $e^-e^+ \to ZAA$

Integrating the differential effective cross section (61) at the angles of departure of particles, for the energy distribution of CP-odd Higgs bosons we obtain the formula

$$\frac{d\sigma}{dx_1 dx_2} = \frac{\sqrt{2}G_F^3 M_Z^6}{48\pi^3 s} \cdot \frac{g_L^2 + g_R^2}{(1 - r_Z)^2}\psi, \tag{62}$$

where

$$\psi = a^2 r_Z F_0 + \frac{a}{2}\left[\frac{\cos^2(\beta-\alpha)}{y_1+r_A-r_h}+\frac{\sin^2(\beta-\alpha)}{y_1+r_A-r_H}\right]^2 F_1$$
$$+\frac{a}{2}\left[\frac{\cos^2(\beta-\alpha)}{y_2+r_A-r_h}+\frac{\sin^2(\beta-\alpha)}{y_2+r_A-r_H}\right]^2 F_2$$
$$+\frac{1}{2r_Z}\left[\frac{\cos^2(\beta-\alpha)}{y_1+r_A-r_h}+\frac{\sin^2(\beta-\alpha)}{y_1+r_A-r_H}\right]$$
$$\times\left[\frac{\cos^2(\beta-\alpha)}{y_2+r_A-r_h}+\frac{\sin^2(\beta-\alpha)}{y_2+r_A-r_H}\right] F_3 \quad (63)$$
$$+\frac{1}{4r_Z}\left[\frac{\cos^2(\beta-\alpha)}{y_1+r_A-r_h}+\frac{\sin^2(\beta-\alpha)}{y_1+r_A-r_H}\right]^2 F_4$$
$$+\frac{1}{4r_Z}\left[\frac{\cos^2(\beta-\alpha)}{y_2+r_A-r_h}+\frac{\sin^2(\beta-\alpha)}{y_2+r_A-r_H}\right]^2 F_5,$$

$$F_0 = \frac{1}{4}\left[(y_1+y_2)^2 - 8r_Z\right],$$
$$F_1 = y_1(y_1-1)(r_Z-y_1) - y_1(y_1+1)(y_1+r_Z) + 2r_Z(1+r_Z-4r_A),$$
$$F_2 = y_2(y_2-1)(r_Z-y_2) - y_2(y_2+1)(y_1+r_Z) + 2r_Z(1+r_Z-4r_A),$$
$$F_3 = \left[r_Z(1+r_Z-y_1-y_2-8r_A)-(1+r_Z)y_1y_2\right](2+2r_Z-y_1-y_2)$$
$$+ y_1y_2\left[y_1y_2+r_Z^2+1+4r_A(1+r_Z)\right]+4r_Ar_Z(1+r_Z+4r_A)-r_Z^2, \quad (64)$$
$$F_4 = (y_1-1)^2(r_Z-y_1)^2 - 4r_Ay_1(y_1+r_Zy_1-4r_Z)$$
$$+ r_Z(r_Z-4r_A)(1-4r_A)-r_Z^2,$$
$$F_5 = (y_2-1)^2(r_Z-y_2)^2 - 4r_Ay_2(y_2+r_Zy_2-4r_Z)$$
$$+ r_Z(r_Z-4r_A)(1-4r_A)-r_Z^2.$$

Figure 10 shows the dependence of the differential effective cross-section of the reaction $e^-e^+ \to ZAA$ on the scaling energy x_1 at a fixed $x_2 = 0.925$. With the growth of the variable x_1, the effective cross-section increases.

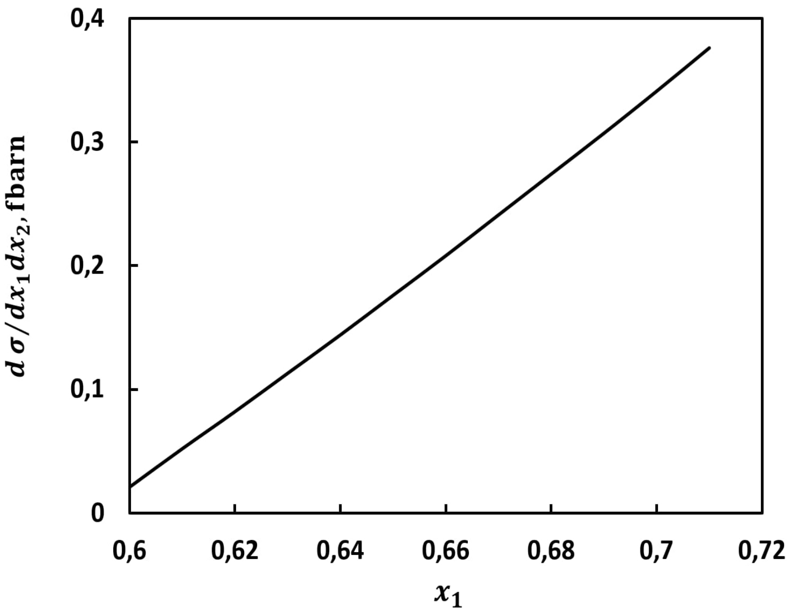

Figure 10: Energy dependence of the cross section of the reaction $e^-e^+ \to ZAA$.

4 Conclusion

Thus, we investigate the process of the production of three CP-odd Higgs bosons during the annihilation of an arbitrarily polarized electron-positron pair $e^-e^+ \to AAA$ and the process of the joint production of a vector Z-boson and two CP-odd Higgs bosons $e^-e^+ \to ZAA$. Taking into account all Feynman diagrams a), b), c) Fig. 1 and diagrams a), b), c), d) Fig. 5, analytical expressions of differential cross sections of processes are obtained. Left-right A_{LR} and transverse A_φ spin asymmetries due to longitudinal and transverse polarizations of the electron-positron pair are determined. The dependence of spin asymmetries and differential effective cross sections on the departure angles and particle energies is studied in detail.

Note that the experimental study of the reactions of the production of three CP-odd Higgs bosons and the reaction of the production of a vector Z-boson and two CP-odd Higgs bosons is of particular interest, since it allows you to accurately measure the constants of the three-boson interactions λ_{hAA} and λ_{HAA}, the constant of the interactions of CP-even Φ ($\Phi = h$ or H) and two CP-odd Higgs bosons $g_{\Phi AA}$.

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