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MARKOV MODEL OF QUEUEING SYSTEM WITH INSTANTANEOUS FEEDBACK AND SERVER SETUP TIME

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Abstract. The Markovian model of single server queueing system with instantaneous feedback is considered. After completion of the service, each call in accordance to Bernoulli scheme either leave the system or immediately feedback for re-service. It is assumed that positive random server setup time is required to start the service of feedback call (f-call). Rate of primary calls (p-calls) depend on server status, i.e. server might be in either working mode or setup mode. Ergodicity condition for the appropriate two-dimensional Markov chain is established and matrix-geometric approach is used to calculate its steady-state probabilities. Minimization of total cost is performed. Results of numerical experiments are demonstrated.

Keywords: queueing systems, instantaneous feedback, positive server setup time, matrix-geometric method, optimization.

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1 Introduction

The history of the study of Queueing Systems with Feedback (QSwFB) originates from the pioneering works of Takacs (Takacs, 1963, 1977). They are useful tools to study telecommunication networks, computer systems, inventory systems and so on. This is explained by the fact that taking into account the feedback phenomena make it possible to increase the adequacy of the models to a real situation where once serviced calls may require either single or multiple re-servicing. For instance, telecommunication system will re-transmit data when the data transmission error occurs. Another example is manufacturing systems where defective parts need to be re-worked. A third example is queueing-inventory systems in which a customer who has received service of high quality once can turn to them again for service (Melikov et al., 2019b).

In class of QSwFB are distinguished three kind of models: models with only instantaneous feedback (QSwIFB), models with only delayed feedback (QSwDFB) and models with both types of feedback (QSwIDFB). All kinds of models are intensively investigated in last three decades. Detailed review of available works is given in Melikov et al. (2015). We also note some recent papers Jain & Kaur (2020), Laxmi & Kassahun (2017), Melikov et al. (2019a), Niranjan et al. (2017), Som & Seth (2018), Varalakshmi et al. (2016) that are not included in the reference list of indicated work Melikov et al. (2015).

Common assumption in most of the papers related to QSwFB is following one: server can immediately start the service of feedback calls (f-calls). However, this assumption is not realistic one in systems where some positive server setup time is required to start the re-servicing of f-calls. Such situations are common ones in manufacturing systems,where server take some “warm-up” (setup) time to process a defective part.

However,there are some papers which consider set up time required by server for restarting service. Choudhury (2000) studied the steady state behaviour of the queue size distributions of an $M^X/G/1$ queueing system with a vacation period which involves an idle period and a random setup period. Ayyappan& Shyamala (2016) analyzed an $M[X]/G/1$ queue with Poisson arrivals, customer feedback, random server breakdowns, Bernoulli schedule server vacation and a random setup time at the end of a busy period. They obtained the probability generating function in terms of Laplace transforms and the corresponding steady state results explicitly.

Here the model of QSwIFB with positive server setup time is investigated. The model with common queue for both primary (p-calls)and f-calls is considered. Similar model by using probability generating function method and space merging approach was investigated in Melikov et al. (2020). Also, an average number of calls in system and rate of served calls are calculated and accuracy of developed approximate space merging method is proved via numerical experiments. Here in this paper, other performance measures are calculated by using matrix-geometric method Neuts (1981) and optimization problem is considered. Goal of optimization problem is minimization of expected total cost related both to providing service in various regimes (i.e. in working and setup regimes) and switching from setup regime to working regime. Note that the proposed here model adequately describes the system with server vacation, where vacation is done after completion of each service act independently on length of queue. Besides, it can be serve as model of the system with unreliable server in which failure of server might be occur only at completion of service.

The paper has the following structure. In next Section 2 the investigated model is described and appropriate level independent quasi birth-death (LIQBD) process is constructed. In Section 3 an ergodicity condition is established and algorithm to calculate the steady-state probabilities of constructed LIQBD has developed. Formulas for calculation of desired performance measures are obtained in Section 4. Optimization problem is formulated in Section 5. Results of numerical experiments are demonstrated in Section 6. Conclusion remarks are given in Section 7.

Notations and abbreviations used in the sequel:

- $\boldsymbol{e}$: Column vector of $1'$s of appropriate order.
- $CTMC$: Continuous time Markov chain.
- I: identity matrix of appropriate order.
- $LIQBD$: Level independent Quasi-Birth and-Death

2 Model Description and Mathematical Formulation

Consider the single server system with infinite queue and instantaneous feedback (IFB). After completion of service, each call according Bernoulli trials either with probability σ depart the system or with probability $1-\sigma$ instantaneously feed back to system for additional service (IFB). Server setup time for starting service of IFB-call is exponentially distributed positive random variable (r.v.) with parameter θ. In other words, server might be in two status: working regime and setup regime. Service time of both type of calls: primary (from outside) and feedback are r.v. with common exponential distribution function with average μ. Calls that arrive from outside have information about the status of server, i.e. when server is in working regime then intensity of input Poisson flow is λ_1 while when server is in setup regime then intensity is λ_0. Server setup time does not get interrupted. Each call, independently on server status, joins the infinite queue. It is required to find the steady-state distribution of the system and its main performance measures.

2.1 The QBD process

The model described in section 1 can be studied as a LIQBD process. First we introduce the following notations:
At time t:
$N(t)$: number of customers in the system,

$$K(t) = \begin{cases} 0, & if\ the\ server\ is\ in\ setup\ regime \\ 1, & if\ the\ server\ is\ in\ working\ regime \end{cases}$$

It is easy to verify that $\{(N(t), K(t)) : t \geq 0\}$ is a LIQBD with state space

$$E = E_0 \cup \bigcup_{i=1}^{\infty} E_i$$

where $E_0 = \{(0, 1)\}$ and for $n \geq 1$, $E_n = \{(n, 0), (n, 1)\}$.

The infinitesimal generator of this CTMC is

$$\mathcal{Q} = \begin{bmatrix} B_0 & C_0 & & & \\ B_1 & A_1 & A_0 & & \\ & A_2 & A_1 & A_0 & \\ & & \ddots & \ddots & \ddots \end{bmatrix}.$$

Here the matrix B_0 is a 1×1 matrix. C_0 is of order 1×2 and B_1 is of order 2×1. A_0 , A_1 , and A_2 are square matrices of order 2.

Define the entries $(B_0)_{k_1}^{k_2}$, $(C_0)_{k_1}^{k_2}$ and $(B_1)_{k_1}^{k_2}$ as transition submatrices which contain transitions of the form $(0, k_1) \to (0, k_2)$, $(0, k_1) \to (1, k_2)$ and $(1, k_1) \to (0, k_2)$, respectively.

Define the entries $(A_0)_{k_1}^{k_2}$,$(A_1)_{k_1}^{k_2}$ and $(A_2)_{k_1}^{k_2}$ as transition submatrices which contains transitions of the form $(n, k_1) \to (n + 1, k_2)$, where $n \geq 1$; $(n, k_1) \to (n, k_2)$, for $n \geq 1$ and $(n, k_1) \to (n - 1, k_2)$, with $n \geq 2$ respectively.

Since none or one event alone could take place in a short interval of time with positive probability, in general a transition such as $(n_1, k_1) \to (n_2, k_2)$ has positive rate only for exactly one of n_1, k_1 different from n_2, k_2.

$$(B_0)_{k_1}^{k_2} = \{ \ -\lambda_1 \quad k_1 = k_2 = 1$$

$$(C_0)_{k_1}^{k_2} = \begin{cases} \lambda_1 & k_1 = k_2 = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$(B_1)_{k_1}^{k_2} = \begin{cases} \mu\sigma & k_1 = k_2 = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$(A_0)_{k_1}^{k_2} = \begin{cases} \lambda_0 & k_1 = k_2 = 0 \\ \lambda_1 & k_1 = k_2 = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$(A_1)_{k_1}^{k_2} = \begin{cases} -(\theta + \lambda_0) & k_1 = k_2 = 0 \\ \theta & k_1 = 0, k_2 = 1 \\ \mu(1 - \sigma) & k_1 = 1, k_2 = 0 \\ -(\mu + \lambda_1) & k_1 = k_2 = 1 \end{cases}$$

$$(A_2)_{k_1}^{k_2} = \begin{cases} \mu\sigma & k_1 = k_2 = 1 \\ 0 & \text{otherwise} \end{cases}$$

Next we proceed for the steady state analysis of the system described.

3 Steady State Analysis

To this end we first obtain the

3.1 Stability condition

Let $\boldsymbol{\pi} = (\boldsymbol{\pi_0}, \boldsymbol{\pi_1})$ denote the steady state probability vector of the generator

$$A = A_0 + A_1 + A_2 = \begin{bmatrix} -\theta & \theta \\ \mu(1-\sigma) & -\mu(1-\sigma) \end{bmatrix}.$$

ie,

$$\boldsymbol{\pi} A = 0, \boldsymbol{\pi} e = 1. \tag{1}$$

Solving (1), we get,

$$\pi_0 = \frac{\mu(1-\sigma)}{\theta + \mu(1-\sigma)} \tag{2}$$

and

$$\pi_1 = \frac{\theta}{\theta + \mu(1-\sigma)}. \tag{3}$$

The *LIQBD* description of the model indicates that the queueing system is stable (see (Neuts, 1981)) if and only if the left drift exceeds that of right drift. That is,

$$\boldsymbol{\pi} A_0 \mathbf{e} < \boldsymbol{\pi} A_2 \mathbf{e}. \tag{4}$$

The inequality (4) is simplified in (5) below.

Thus we arrive at the following.

Proposition 1. *The system is stable iff*

$$\mu(1-\sigma)\lambda_0 + \theta\lambda_1 < \theta\mu\sigma. \tag{5}$$

Remark 1. *When $\sigma = 1$, the stabilty condition becomes*

$$\lambda_1 < \mu.$$

The model reduces to classical case when $\sigma = 1$.

3.2 Steady-state probability vector

Assuming that the condition (5) is satisfied we proceed to find the steady-state probability of the system state. Let $\boldsymbol{x}$ be the steady state probability vector of Q.We partition this vector as $\boldsymbol{x} = (\boldsymbol{x_0}, \boldsymbol{x_1}, \boldsymbol{x_2} \ldots)$, where $\boldsymbol{x_0}$ is of dimension 1 and $\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots$ are each of dimension 2 . Under the stability condition, we have $\boldsymbol{x_i} = \boldsymbol{x}_1 R^{i-1}, i \geq 2$, where the matrix R is the minimal nonnegative solution to the matrix quadratic equation

$$R^2 A_2 + R A_1 + A_0 = 0$$

and the vectors $\boldsymbol{x_0}$ and $\boldsymbol{x_1}$are obtained by solving the equations

$$\boldsymbol{x_0} B_0 + \boldsymbol{x_1} B_1 = 0 \tag{6}$$

$$\boldsymbol{x_0} C_0 + \boldsymbol{x_1}(A_1 + R A_2) = 0 \tag{7}$$

subject to the normalizing condition

$$\boldsymbol{x_0} \boldsymbol{e} + \boldsymbol{x_1}(I - R)^{-1} \boldsymbol{e} = 1. \tag{8}$$

4 Some Important Performance Measures

- The probability that the server is idle:

$$p_{idle} = x_0$$

- Mean number of customers in the system:

$$E_s = \sum_{n=1}^{\infty} \sum_{k=0}^{1} n x_{n,k}$$

- Mean number of customers in the system when the server is in setup regime:

$$E_{sr} = \sum_{n=1}^{\infty} n x_{n,0}$$

- Mean number of customers in the system when the server is in working regime:

$$E_{wr} = \sum_{n=1}^{\infty} n x_{n,1}$$

- The fraction of time the server is in setup regime:

$$T_{sr} = \sum_{n=1}^{\infty} x_{n,0}$$

- The fraction of time the server is in working regime:

$$T_{wr} = \sum_{n=0}^{\infty} x_{n,1}$$

- Expected rate of switching of server to working regime:

$$R_{sw} = \sum_{n=1}^{\infty} x_{n,0} \theta$$

5 Analysis of a cost function

We construct a cost function based on the above performance measures.

Let
C_h:Unit time cost of holding a customer
C_{sr}: Unit time cost of providing service when the server is in setup regime
C_{wr}: Unit time cost of providing service when the server is in working regime
C_{sw}: Unit time cost of switching to working regime
Then the expected cost per unit time,

$$C = E_s \times C_h + T_{sr} \times C_{sr} + T_{wr} \times C_{wr} + R_{sw} \times C_{sw}$$

6 Numerical Results

In this section we study the effect of different parameters on various performance measures and the cost function.

First we fix the parameters as $\lambda_1 = 1.2, \mu = 10, \sigma = 0.6, \theta = 2, C_h = 3, C_{sr} = 2, C_{wr} = 100$, and $C_{sw} = 10$ to study the effect of λ_0.

Table 1: Effect of λ_0: Fix $\lambda_1 = 1.2, \mu = 10, \sigma = 0.6, \theta = 2, C_h = 3, C_{sr} = 2, C_{wr} = 100$ and $C_{sw} = 10$

λ_0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
E_s	0.5077	0.5996	0.7372	0.9389	1.2354	1.6800	2.3712	3.5107	5.5776	10.0273
T_{sr}	0.3000	0.3158	0.3333	0.3529	0.3750	0.4000	0.4286	0.4615	0.5000	0.5455
T_{wr}	0.7000	0.6842	0.6667	0.6471	0.6250	0.6000	0.5714	0.5385	0.5000	0.4545
R_{sw}	0.6000	0.6316	0.6667	0.7059	0.7500	0.8000	0.8571	0.9231	1.000	1.0909
C	78.1232	77.1674	76.2115	75.2872	74.4562	73.8400	**73.6850**	74.5321	77.7329	87.5364

From Table 1, we see that E_s and T_{sr} increases when λ_0 increases, as expected. As a result T_{wr} decreases. As λ_0 increases, the expected number of customers in the system when the server is in set up regime increases and as a result R_{sw} also increases. The cost function first decreases to a minimum value and after that it increases.

Next, we fix the parameters as $\lambda_0 = 1, \mu = 10, \sigma = 0.6, \theta = 2, C_h = 3, C_{sr} = 2, C_{wr} = 35$, and $C_{sw} = 10$ to study the effect of λ_1.

Table 2: Effect of λ_1: Fix $\lambda_0 = 1, \mu = 10, \sigma = 0.6, \theta = 2, C_h = 3, C_{sr} = 2, C_{wr} = 35$ and $C_{sw} = 10$

λ_1	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
E_s	0.2275	0.4361	0.6345	0.8296	1.0278	1.2354	1.4593	1.7074	1.9897	2.3194
T_{sr}	0.0909	0.1667	0.2308	0.2857	0.3333	0.3750	0.4118	0.4444	0.4737	0.5000
T_{wr}	0.9091	0.8333	0.7692	0.7143	0.6667	0.6250	0.5882	0.5556	0.5263	0.5000
R_{sw}	0.1818	0.3333	0.4615	0.5714	0.6667	0.7500	0.8235	0.8889	0.9474	1.000
C	34.5006	34.1417	33.9034	33.7744	**33.7500**	33.8312	34.0250	34.3444	34.8112	35.4583

From Table 2, we see that E_s increases when λ_1 increases as expected. T_{wr} decreases and T_{sr} increases as λ_1 increases, since a customer gets service when the server is in working regime. As λ_1 increases, the expected number of customers in the system when the server is in set up regime increases and as a result R_{sw} also increases. The cost function first decreases to a minimum value and after that it increases.

Next, we fix the parameters as $\lambda_0 = 1, \lambda_1 = 1.2, \sigma = 0.6, \theta = 2, C_h = 3, C_{sr} = 2, C_{wr} = 35$, and $C_{sw} = 10$ to study the effect of μ.

Table 3: Effect of μ: Fix $\lambda_0 = 1, \lambda_1 = 1.2, \sigma = 0.6, \theta = 2, C_h = 3, C_{sr} = 2, C_{wr} = 35$ and $C_{sw} = 10$

μ	3.5	3.6	3.7	3.8	3.9	4	4.1	4.2	4.3	4.4
E_s	10.5079	8.7836	7.5574	6.6422	5.9337	5.3698	4.9107	4.5300	4.2094	3.9360
T_{sr}	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750
T_{wr}	0.6250	0.6250	0.6250	0.6250	0.6250	0.6250	0.6250	0.6250	0.6250	0.6250
R_{sw}	0.7500	0.7500	0.7500	0.7500	0.7500	0.7500	0.7500	0.7500	0.7500	0.7500
C	61.6486	56.4757	52.7973	50.0515	47.9262	46.2344	44.8570	43.7149	42.7533	41.9331

From Table 3, we can see that E_s decreases when μ increases, as expected. Also we see that T_{sr}, T_{wr} and R_{sw} remains constant when μ increases. This is due the effect of feed back of customers to the system after service completion. Also, the cost function decreases as μ increases.

Next, we fix the parameters as $\lambda_0 = 1, \lambda_1 = 1.2, \mu = 10, \sigma = 0.6, , C_h = 3, C_{sr} = 2, C_{wr} = 35$, and $C_{sw} = 10$ to study the effect of θ.

Table 4: Effect of θ: Fix $\lambda_0 = 1, \lambda_1 = 1.2, \mu = 10, \sigma = 0.6, C_h = 3, C_{sr} = 2, C_{wr} = 35$ and $C_{sw} = 10$

θ	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
E_s	4.5122	2.7411	1.9465	1.5081	1.2354	1.0519	0.9212	0.8240	0.7492	0.6902
T_{sr}	0.6000	0.5217	0.4615	0.4138	0.3750	0.3429	0.3158	0.2927	0.2727	0.2553
T_{wr}	0.4000	0.4783	0.5385	0.5862	0.6250	0.6571	0.6842	0.7073	0.7273	0.7447
R_{sw}	0.7200	0.7304	0.7385	0.7448	0.7550	0.7543	0.7579	0.7610	0.7636	0.7660
C	35.9365	33.3101	**32.9934**	33.3173	33.8312	34.3843	34.9214	35.4232	35.8841	36.3046

From Table 4, we see that E_s decreases when θ increases. This happens because when θ increases server shifts from setup regime to working regime at a faster rate. Also T_{sr} decreases and T_{wr} increases when θ increases, as expected. R_{sw} increases as θ increases since the shifting rate of server from setup regime to working regime increases. The cost function first decreases to a minimum value and after that it increases.

Next, we fix the parameters as $\lambda_0 = 1, \lambda_1 = 1.2, \mu = 10, \theta = 2, C_h = 3, C_{sr} = 2, C_{wr} = 35$, and $C_{sw} = 10$ to study the effect of σ.

Table 5: Effect of σ: Fix $\lambda_0 = 1, \lambda_1 = 1.2, \mu = 10, \theta = 2, C_h = 3, C_{sr} = 2, C_{wr} = 35$ and $C_{sw} = 10$

σ	0.5	0.54	0.58	0.62	0.66	0.7	0.74	0.78	0.82	0.86
E_s	3.3317	2.0719	1.4428	1.0709	0.8269	0.6550	0.5271	0.4280	0.3486	0.2833
T_{sr}	0.5455	0.4710	0.4051	0.3465	0.2939	0.2466	0.2037	0.1646	0.1289	0.0961
T_{wr}	0.4545	0.5290	0.5949	0.6535	0.7061	0.7534	0.7963	0.8354	0.8711	0.9039
R_{sw}	1.0909	0.9420	0.8103	0.6930	0.5879	0.4932	0.4073	0.3292	0.2578	0.1922
C	37.9042	35.0928	34.0615	33.7081	**33.6595**	33.7594	33.9338	34.1443	34.3704	34.6005

From Table 5, we see that E_s decreases as σ increases. This happens because when σ increases, the departure rate of customers from the system after getting service increases. Also, when σ increases, T_{sr} decreases since shifting rate of server from working regime to setup regime decreases and as a result T_{wr} increases. As a result, R_{sw} decreases. The cost function first decreases to a minimum value and after that it increases.

7 Conclusion

The mathematical model of queueing system with instantaneous feedback is investigated. The matrix-geometric method is used for steady-state analysis of this system and the problem of minimization of total expected cost is solved. Extension of the model discussed to models with more complex distributions of input flow and service time, different service rates of primary and feedback calls etc are subject of our future works.

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